Almost Optimal Distance Oracles for Planar Graphs

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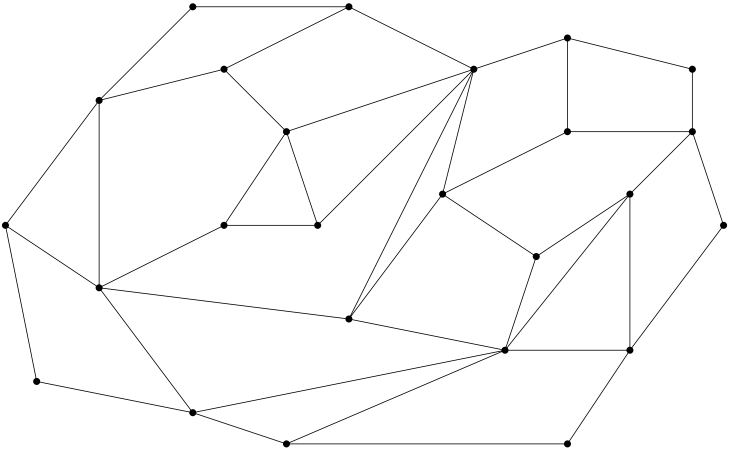
STOC 2019

Problem definition



Preprocess an n-vertex planar graph $G = (V, E)$ with nonnegative arc lengths, so that given any $u, v \in V$ we can compute $d(u, v)$ efficiently.

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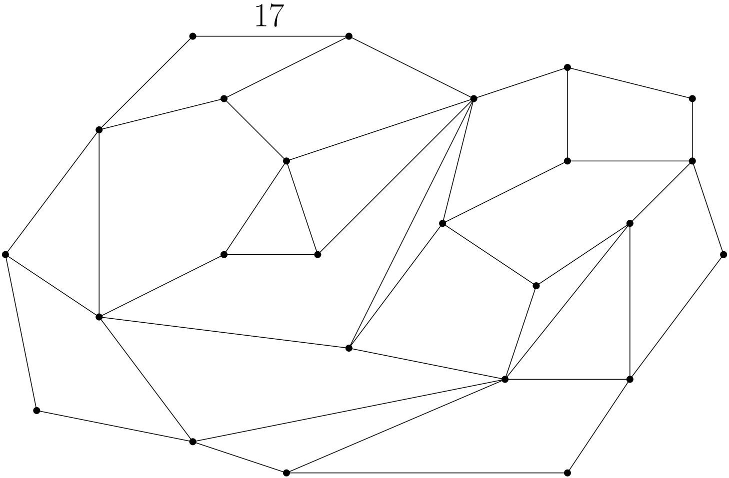

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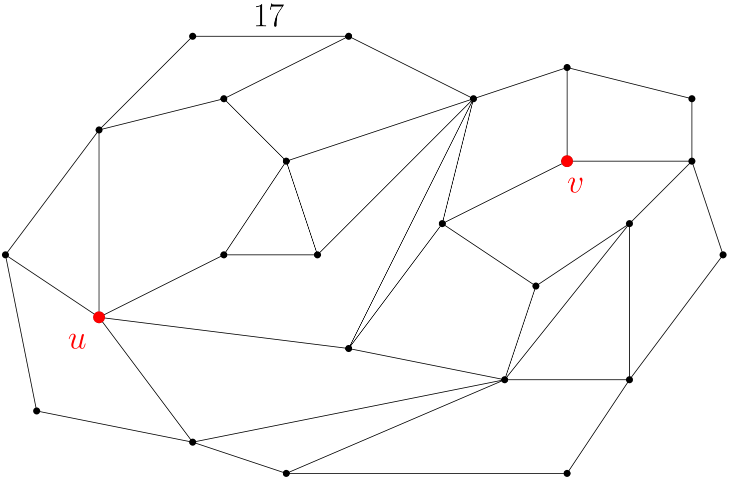

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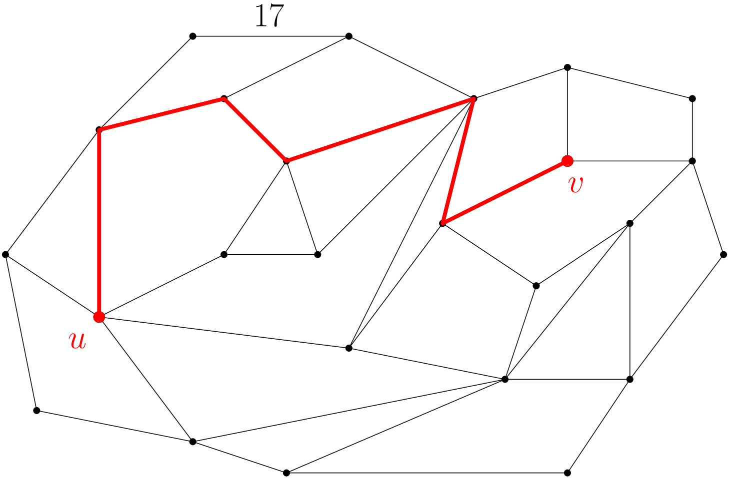

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Goals

We want:

- Fast queries, ideally $Q = O(1)$.
- Small size, ideally $S = O(n)$.
- Fast construction, ideally $T = O(n)$.

The most important tradeoff is between query-time Q and size S.

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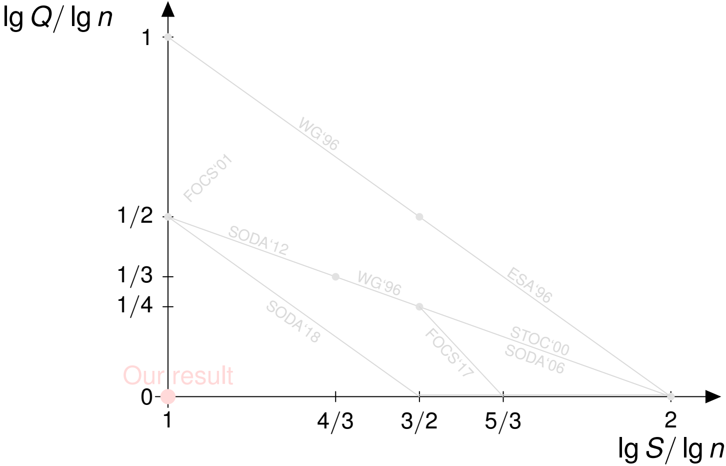
Previous work

The tradeoff between the query-time Q and the size S of the structure:



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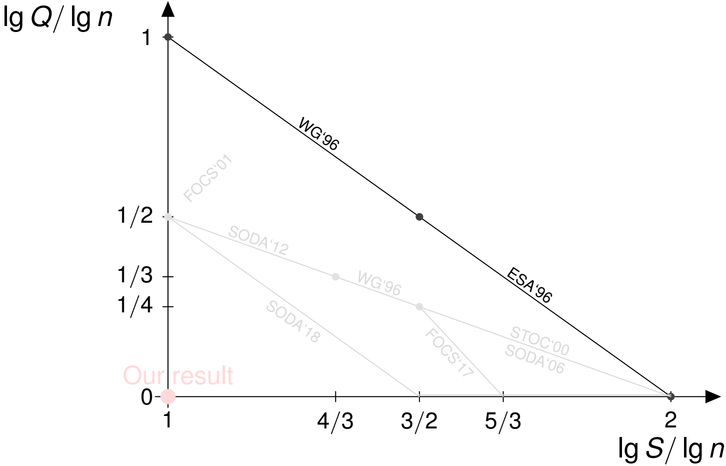

Djidjev and Arikati et al. achieved $Q = O(n^2/S)$.

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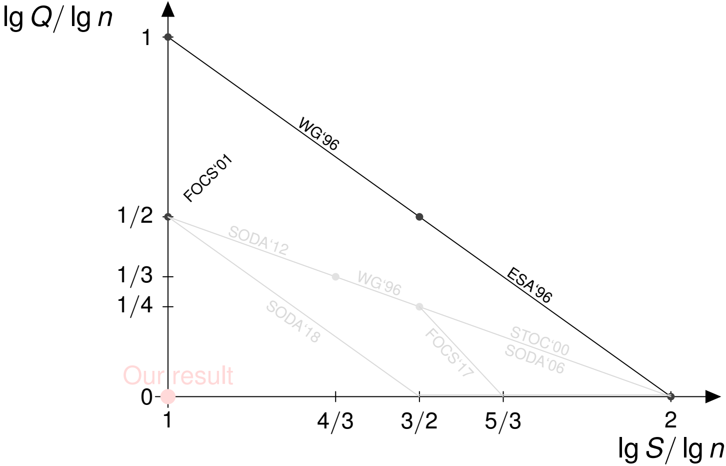

Fakcharoenphol and Rao showed that $S = \tilde{O}(n)$ and $Q = \tilde{O}(\sqrt{n})$ is possible.

Previous work

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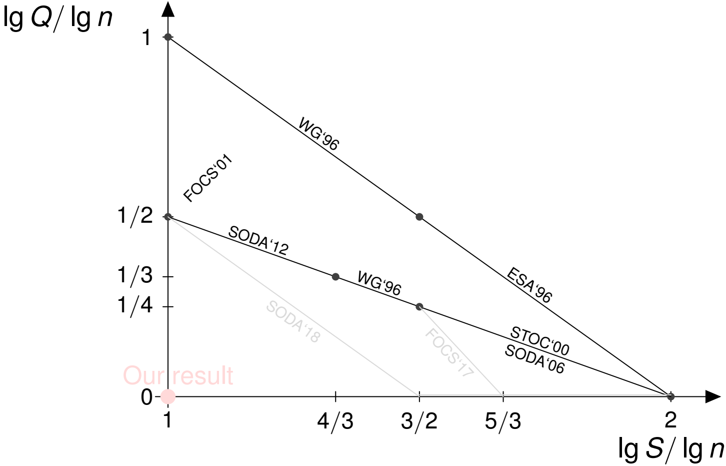

This has been extended to $Q = \tilde{O}(n/\sqrt{S})$ for essentially the whole range of S in a series of papers.

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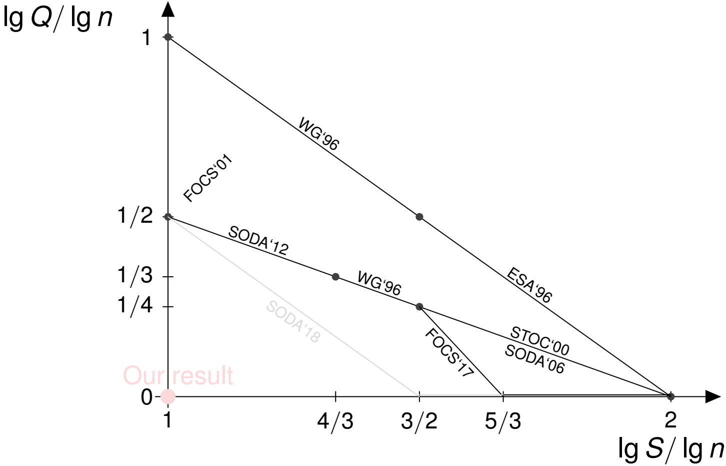

In 2017, Cohen-Addad, Dahlgaard, and Wulff-Nilsen showed that this is not optimal, and $S = O(n^{5/3})$ with $Q = O(\log n)$ is possible.

Previous work

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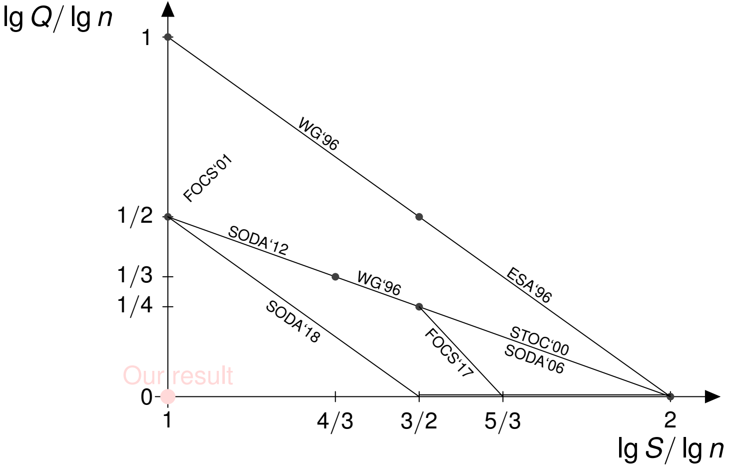

In 2018, Gawrychowski et al. improved this to $S = O(n^{1.5})$ and $Q = O(\log n)$.

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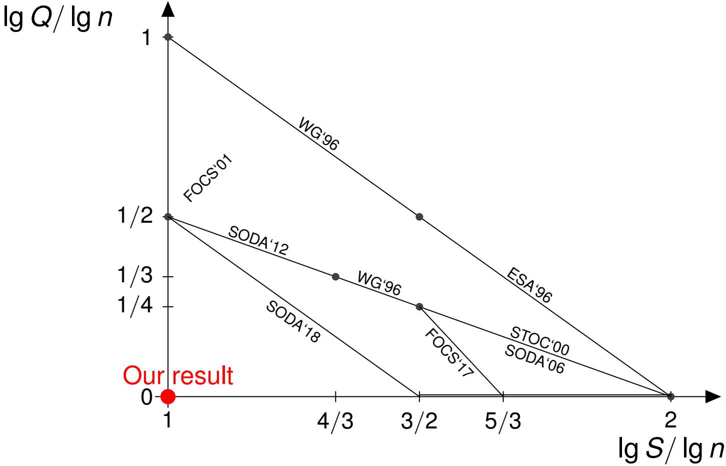

We improve this to $S = O(n^{1+\epsilon})$ and $Q = \tilde{O}(1)$ for any $\epsilon > 0$.

Main result

We show the following tradeoffs for $\langle$space , query-time$\rangle$:

1. $\langle \tilde{O}(n^{1+\epsilon}), O(\log^{1/\epsilon} n)\rangle$, for any constant $1/2 \geq \epsilon > 0$;
2. $\langle O(n \log^{2+1/\epsilon} n), \tilde{O}(n^{\epsilon})\rangle$, for any constant $\epsilon > 0$;
3. $\langle n^{1+o(1)}, n^{o(1)}\rangle$.

The oracle is based on a recursive view of the point location mechanism for Voronoi diagrams of Gawrychowski et al. [SODA'18].

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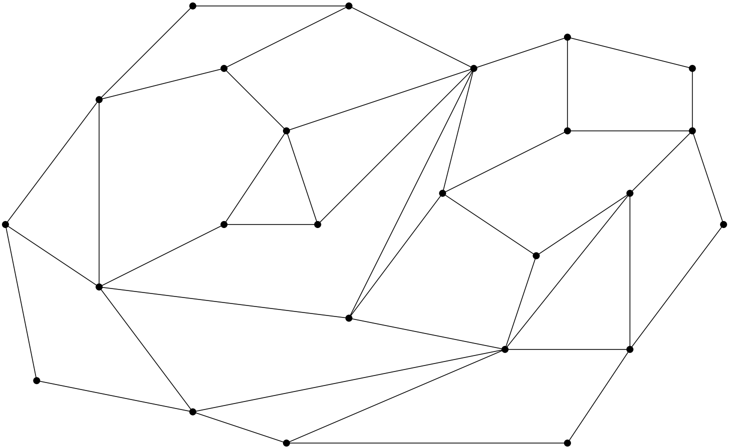
Cycle Separators

Miller [JCSS'86]

There always exists a Jordan curve separator of size $O(\sqrt{n})$ such that there are at most $\frac{2}{3}n$ vertices on its inside/outside.



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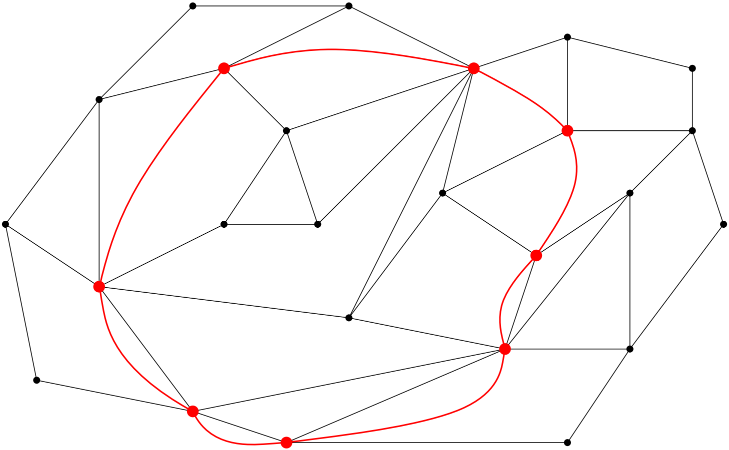
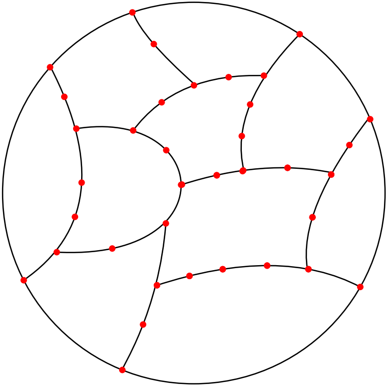


r-divisions

For $r \in [1, n]$, a decomposition of the graph into:

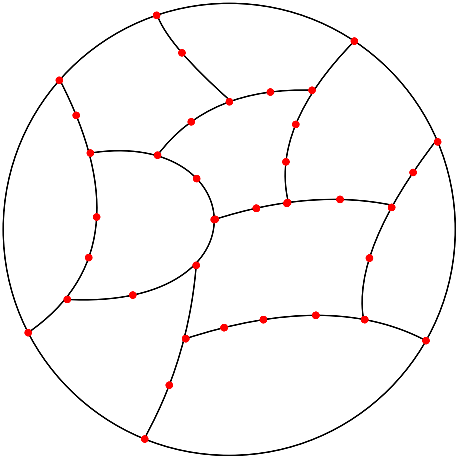
- $O(n/r)$ pieces;
- each piece has $O(r)$ vertices;
- each piece has $O(\sqrt{r})$ boundary vertices (vertices incident to edges in other pieces).



We denote the boundary of a piece P by ∂P and assume that all such vertices lie on a single face of P.

Recursive r-divisions

For $r_1 < r_2 < \cdots < r_m \in [1, n]$, we can efficiently compute r_i-divisions, such that each r_i-division respects the r_{i+1}-division.



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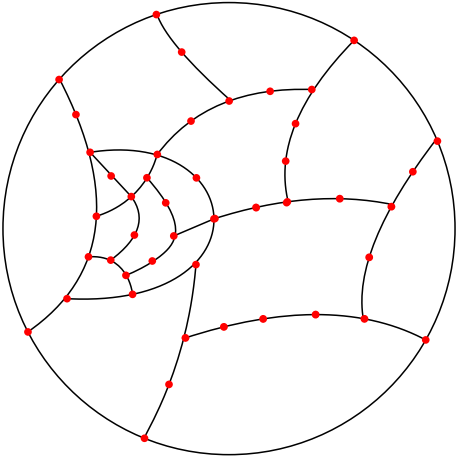
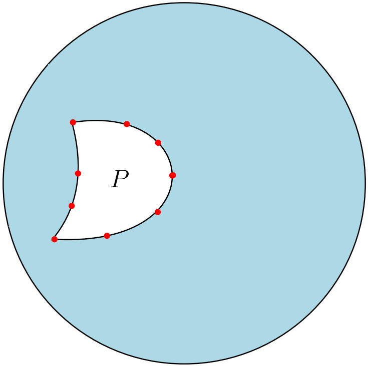


Multiple Source Shortest Paths (MSSP)

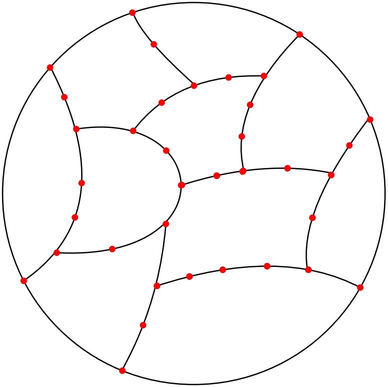
Klein [SODA'05]

There exists a data structure requiring $O(n \log n)$ space that can report in $O(\log n)$ time the distance between any vertex on the infinite face (boundary vertex) and any vertex in the graph.



Warm-up: Space $\tilde{O}(n^{4/3})$, Query-time $\tilde{O}(n^{1/3})$

- Compute an r-division.
- For each vertex $u \in P$, store additive weights $d_G(u,p)$ for $p \in \partial P$.
  Space $O(n \cdot \sqrt{r})$.
- For each piece P, store an MSSP data structure for the outside of P with sources ∂P.
  Space $\tilde{O}(n/r \cdot n)$.



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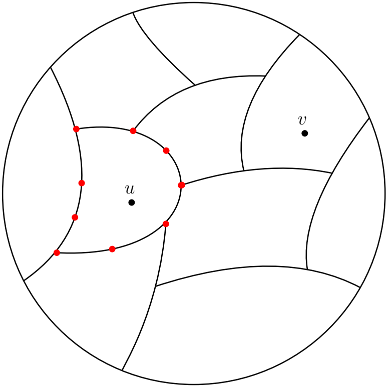

Interesting case.

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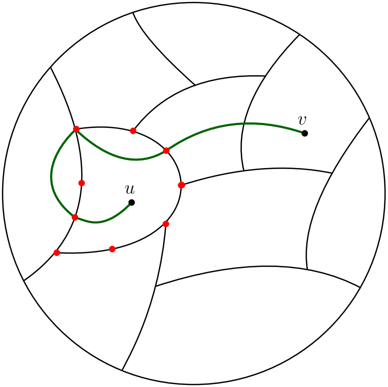

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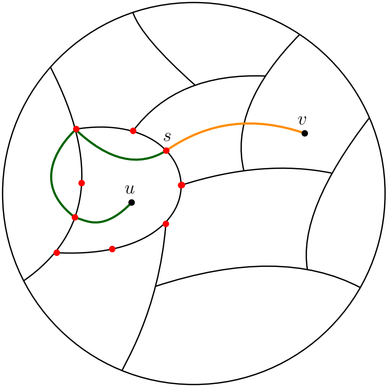

We decompose the path on the last boundary vertex it visits.

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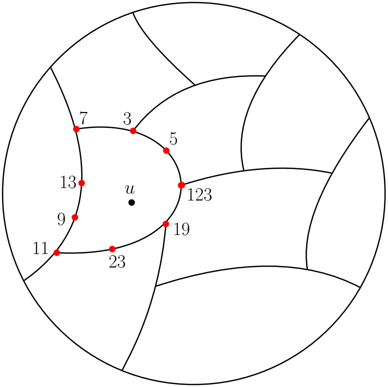

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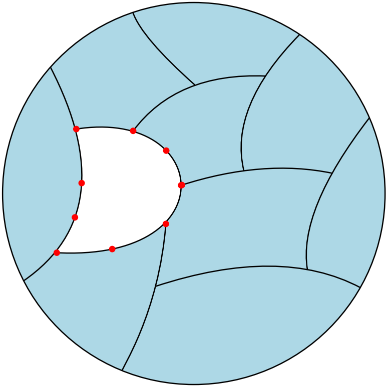

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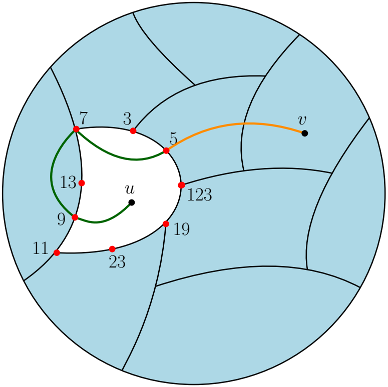

At query, find vertex $s \in \partial P$, minimizing $d_G(u,s) + d_{G\setminus(P\setminus\partial P)}(s,v)$.
This is called *point location*.

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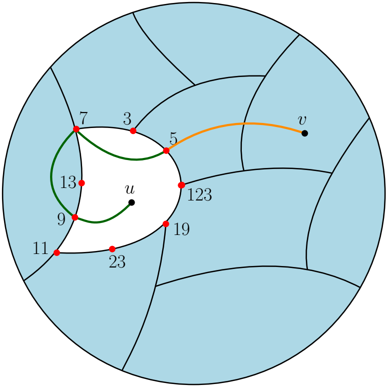

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Perform point location by trying all $O(\sqrt{r})$ boundary vertices.

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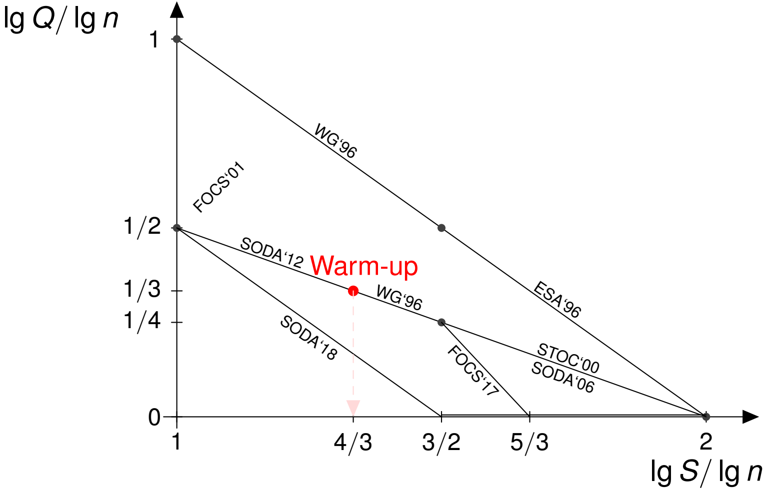

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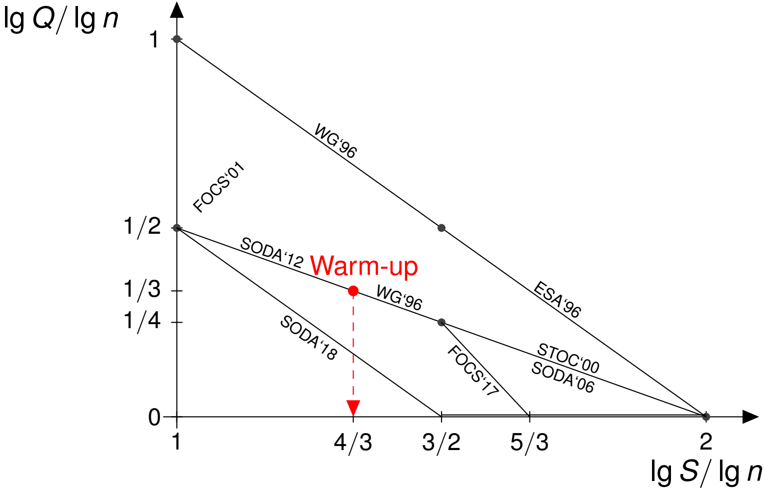
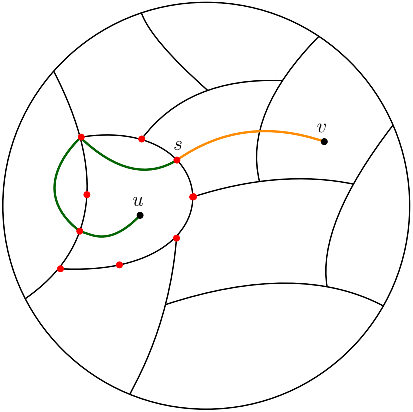


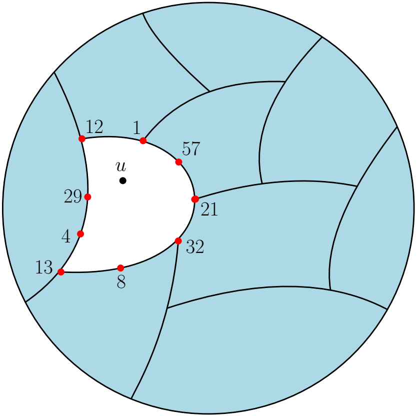
First goal



Instead of trying all possible $O(\sqrt{r}) = O(n^{1/3})$ candidate boundary vertices, we want to compute the last boundary vertex s visited by the shortest path in $\tilde{O}(1)$ time.

Point location

- Each vertex u defines a set of additive weights $d_G(u, p)$ for $p \in \partial P$.



Gawrychowski et. al. [SODA'18]

Given an MSSP data structure for the outside of P, with sources ∂P, there exists an $\tilde{O}(|\partial P|)$-sized data structure for each set of additive weights for ∂P that answers point location queries in $\tilde{O}(1)$ time.

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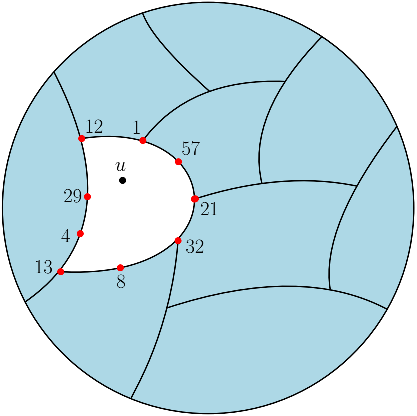

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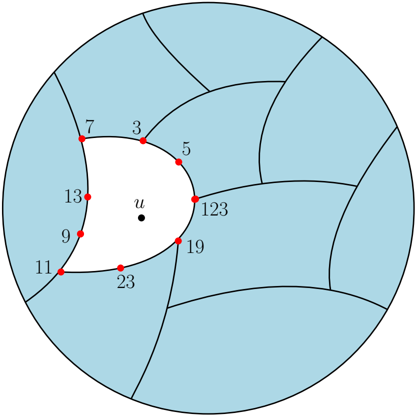

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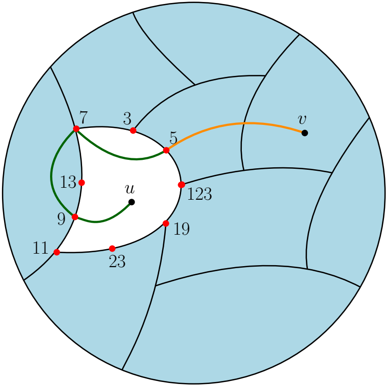
Warm-up 2: Space $\tilde{O}(n^{4/3})$, Query-time $\tilde{O}(1)$

- Compute an r-division.
- For each vertex $u \in P$, store additive weights $d_G(u, p)$ for $p \in \partial P$. Preprocess these for point location.
  Space $O(n \cdot \sqrt{r})$.
- For each piece P, store an MSSP data structure for the outside of P with sources ∂P.
  Space $\tilde{O}(n/r \cdot n)$.



At query, perform point location by trying all possible $O(\sqrt{r})$ candidate boundary vertices.

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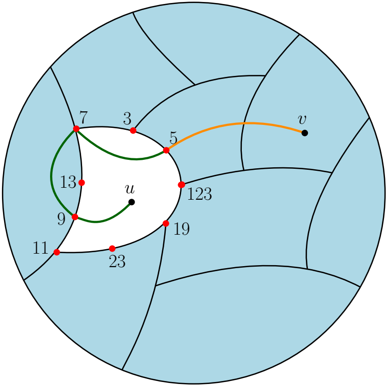

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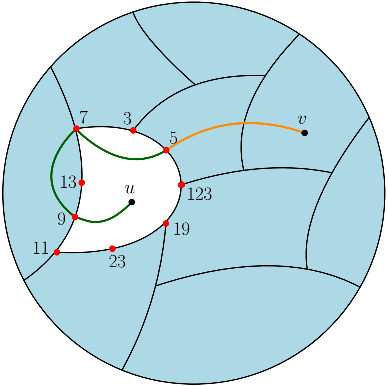

At query, perform point location in $\tilde{O}(1)$ time!

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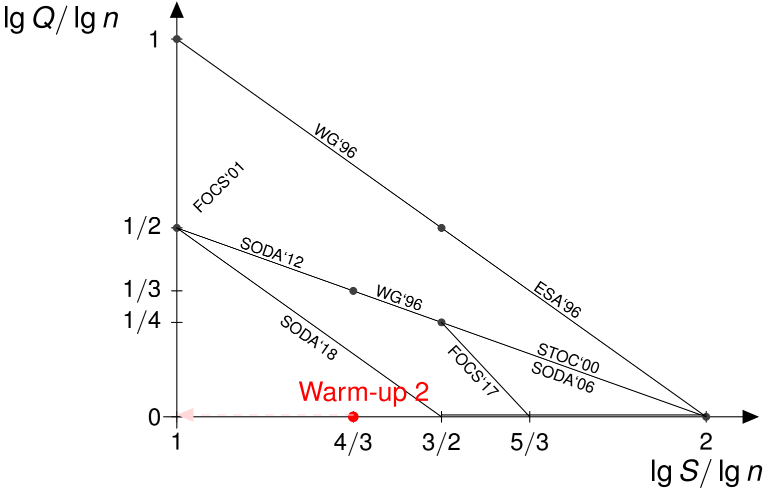

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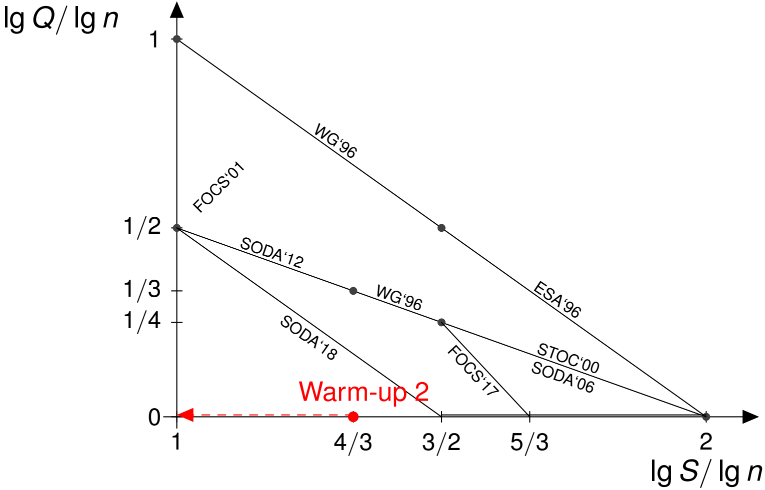

Second goal

Shrink pieces so that cost for storing additive weights shrinks.

- Compute an n^ϵ-division.
- For each vertex $u \in P$, store additive weights $d_G(u, p)$ for $p \in \partial P$. Preprocess these for point location.
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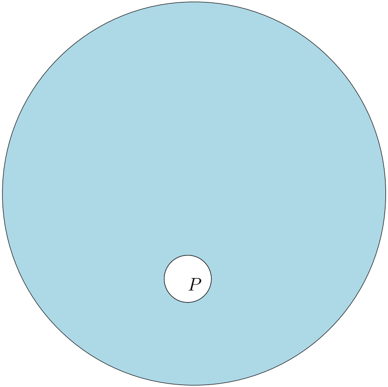

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Shrink pieces so that cost for storing additive weights shrinks.

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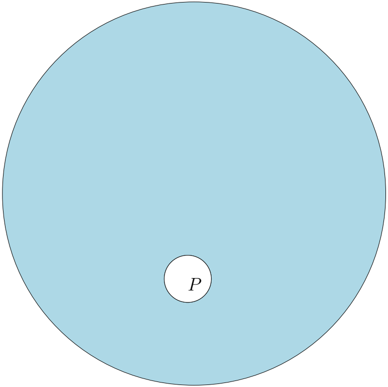

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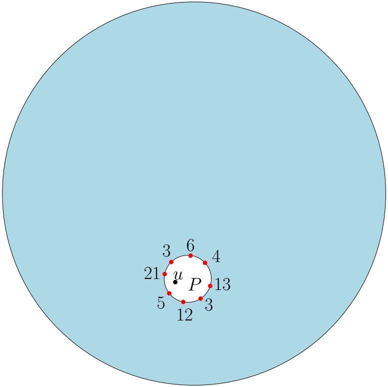

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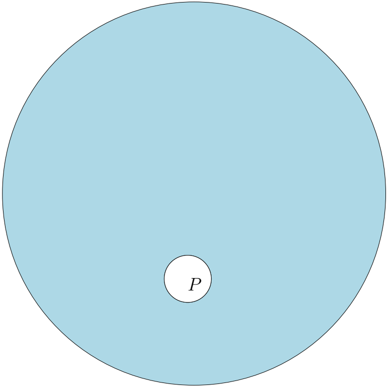
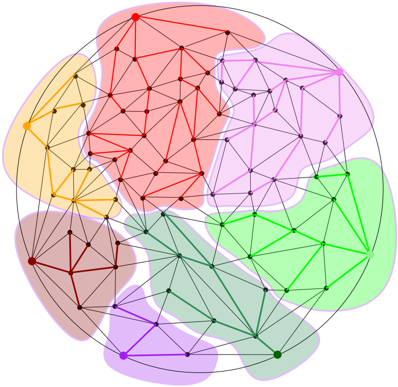
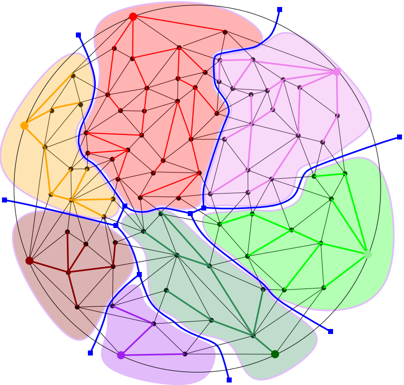


Point Location via Voronoi Diagrams



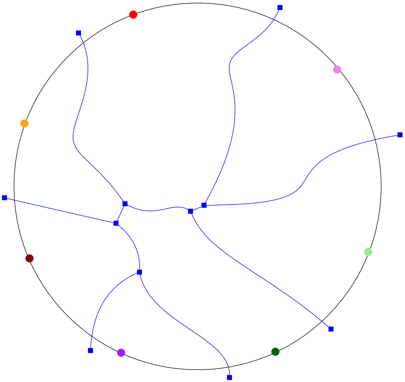
The Voronoi cell of each site consists of all vertices closer to it with respect to the additive distances.

Point Location via Voronoi Diagrams



Because all sites are adjacent to one face, the diagram can be described by a tree on $O(|\partial P|) = O(\sqrt{r})$ vertices (independent of n).

Point Location via Voronoi Diagrams



Any tree on k vertices contains a centroid vertex x such that every component of $T \setminus \{x\}$ is of size $\frac{2}{3}k$.

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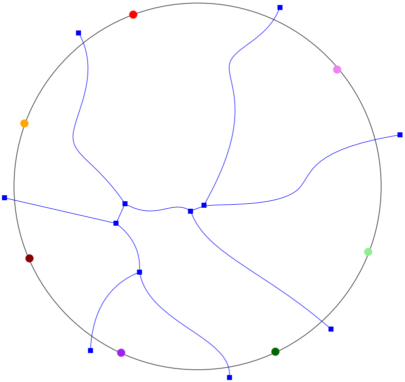

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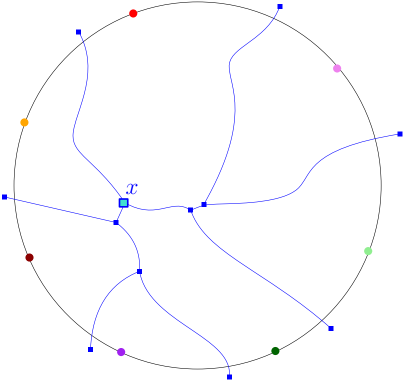
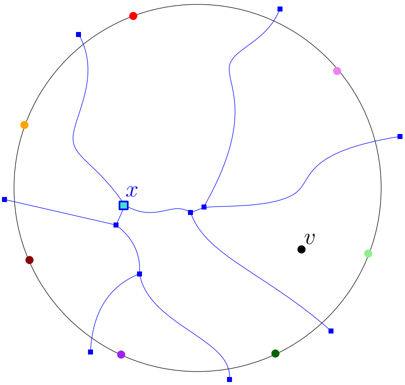


Main idea: Consider the centroid. Find which subtree contains edges adjacent to the Voronoi cell containing v. Recurse.

Point Location via Voronoi Diagrams



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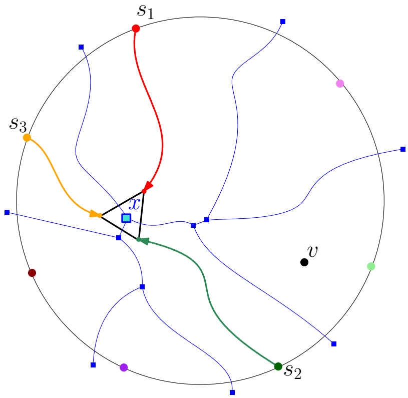
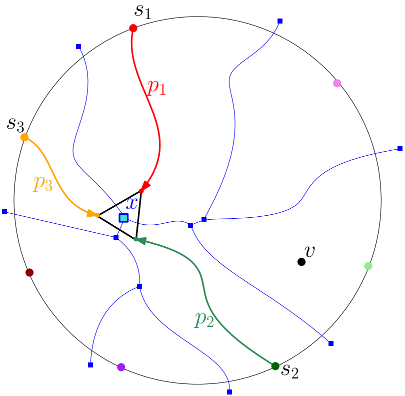


The centroid vertex corresponds to a trichromatic face of the original graph.

Point Location via Voronoi Diagrams



- Find which s_k among s_1, s_2, s_3 is closest to v.
  (distance queries from boundary vertices – MSSP)
- Check whether v is left or right of p_k. (left/right query – MSSP)

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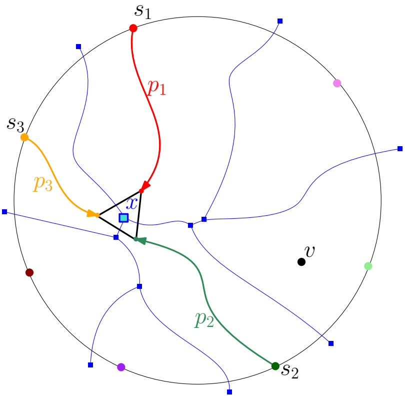

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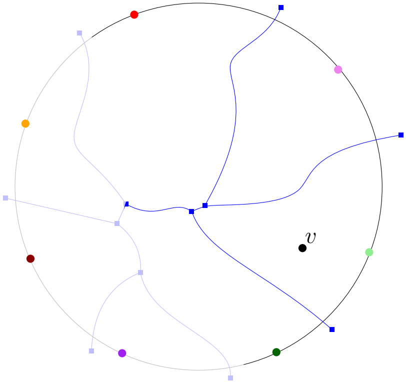

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Idea

Recurse, exploiting the structure of such queries.

Find which s_k among s_1, s_2, s_3 is closest to v.
(distance query from boundary vertices)

Main result: Space $\tilde{O}(n^{1+\epsilon})$, Query-time $\tilde{O}(1)$

- Compute a recursive r-division for $r_i = n^{i \cdot \epsilon}$.
- For each piece P of the n^{ϵ}-division, for each vertex $u \in P$, store a Voronoi diagram for the outside of P with sites ∂P and additive weight $d_G(u, p)$ for $p \in \partial P$. Space $\tilde{O}(n \cdot \sqrt{r_1})$.
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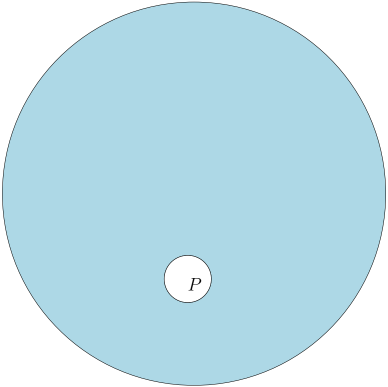

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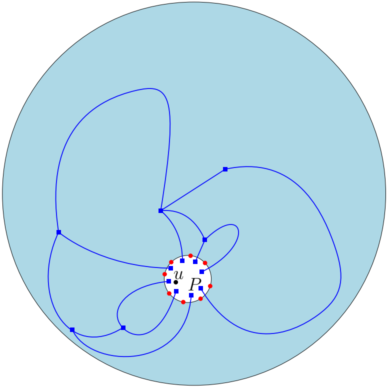

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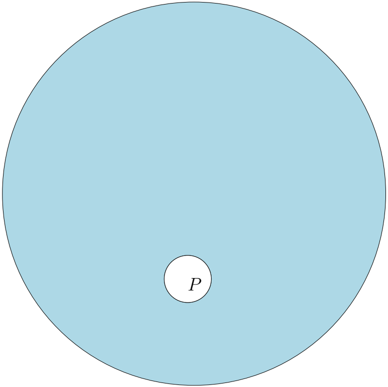

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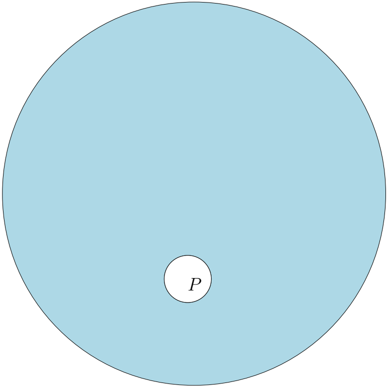

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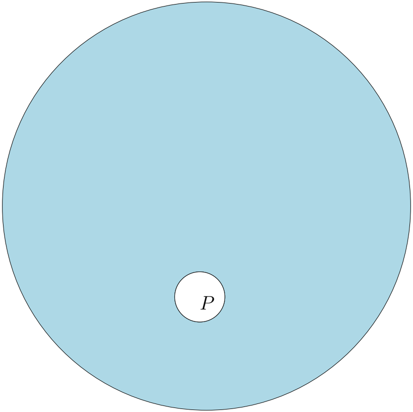
We can not afford to store an $\Omega(n)$-sized MSSP for each of the $n^{1-\epsilon}$ pieces.



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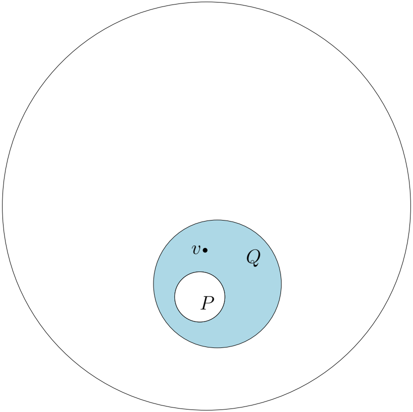
Store an MSSP for piece Q of the $n^{2\cdot\epsilon}$-division that contains P. This handles the case $v \in Q$.
Space: $\tilde{O}(n^{1-\epsilon} \cdot n^{2\epsilon}) = \tilde{O}(n^{1+\epsilon})$.



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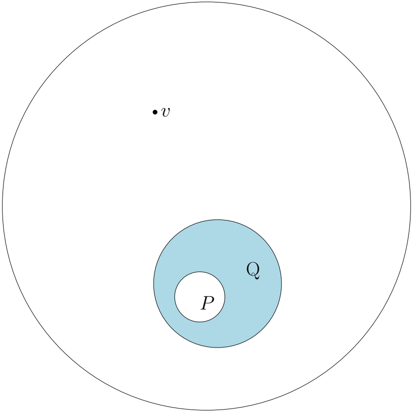

Main result: Space $\tilde{O}(n^{1+\epsilon})$, Query-time $\tilde{O}(1)$

Case $v \notin Q$: each $p \in \partial P$ stores a Voronoi diagram for the outside of Q with sites ∂Q.
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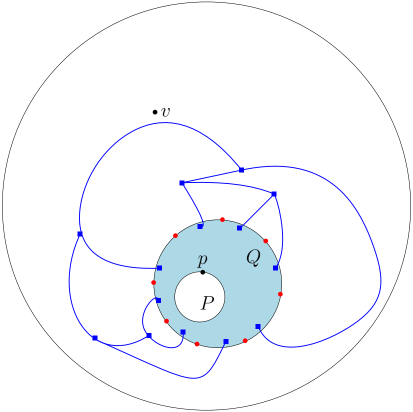

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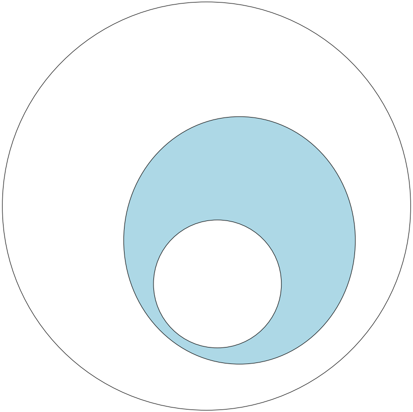
Repeat the same reasoning for increasingly larger pieces of sizes $n^{i\cdot\epsilon}$, for $i = 1, \dots, 1/\epsilon$. There are $n^{1-i\epsilon}$ pieces at level i, each stores MSSP and Voronoi diagrams of size $\tilde{O}(n^{(i+1)\epsilon})$. Total space: $\tilde{O}(\frac{1}{\epsilon}n^{1+\epsilon})$.



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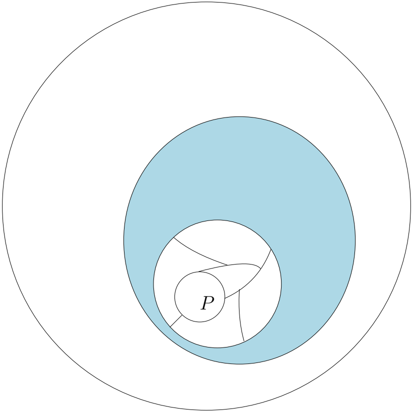
Smaller pieces share the MSSP data structures at higher levels.



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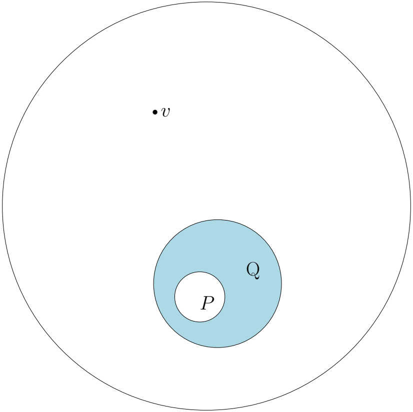
Each point location query, either gets answered at the current level, or reduces to $O(\log n)$ point location queries at a higher level.



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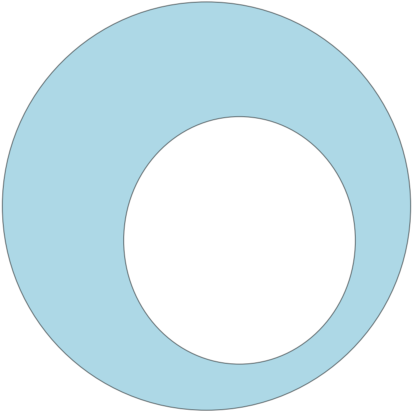
If not earlier, then in the top level we answer the point location query in $O(\log^2 n)$ time.
Query time: $O(\log^{1/\epsilon} n)$.



Tradeoffs and construction time

We show the following tradeoffs for $\langle S, Q \rangle$:

1. $\langle \tilde{O}(n^{1+\epsilon}), O(\log^{1/\epsilon} n) \rangle$, for any constant $1/2 \geq \epsilon > 0$;
2. $\langle O(n \log^{2+1/\epsilon} n), \tilde{O}(n^{\epsilon}) \rangle$, for any constant $\epsilon > 0$;
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Issues and extras:

- left/right queries;
- ∂P is not a single face of P (holes);
- constructing these oracles in $O(n^{3/2+\epsilon})$ time.

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- Improvements on dynamic distance oracles?
  Currently:
  1. exact: UB $\tilde{O}(n^{2/3})$; LB $\tilde{O}(n^{1/2})$ (conditioned on APSP)
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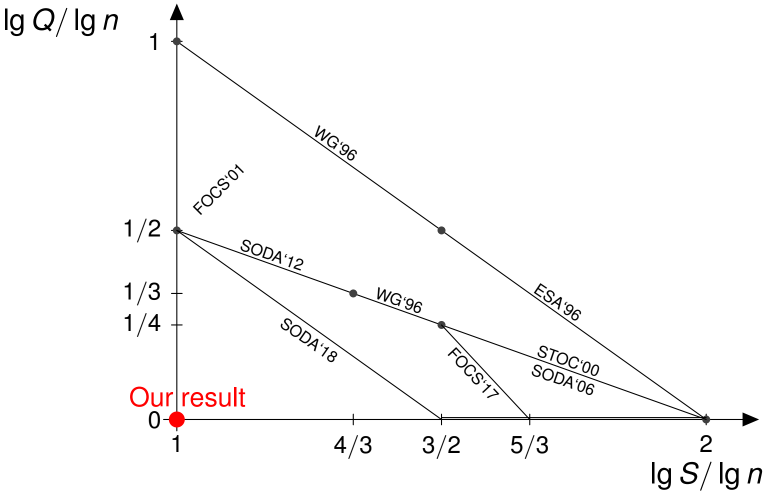

Thanks! Questions?