Internal Dictionary Matching

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[4]Samsung R&D Institute Poland

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Introduction

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space $\mathcal{O}(||\mathcal{D}||)$, $t_{query} = \mathcal{O}(|T| + |output|)$.

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Other internal queries: longest common prefix of two suffixes of T, periods of a substring of T, etc.

IPM → 2D Range Reporting (Mäkinen and Navarro)

T:	a	d	a	a	a	a	b	a	a	b	b	a	a	c
	1	2	3	4	5	6	7	8	9	10	11	12	13	14

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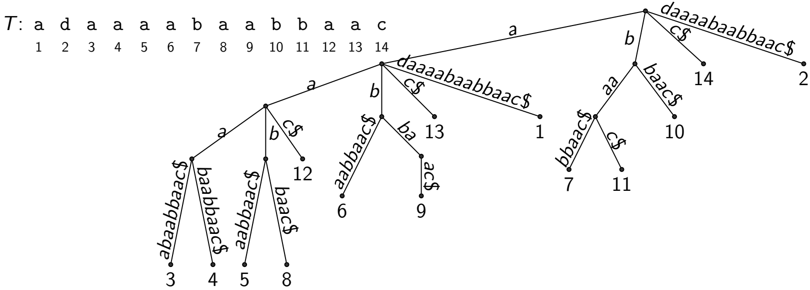

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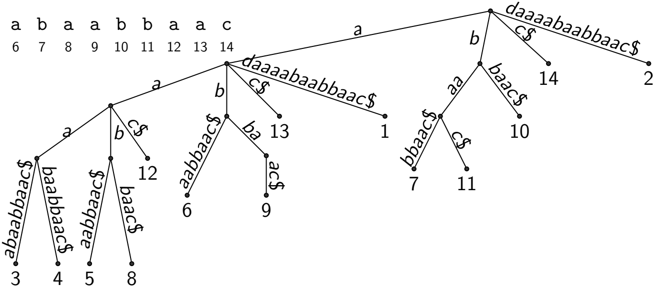

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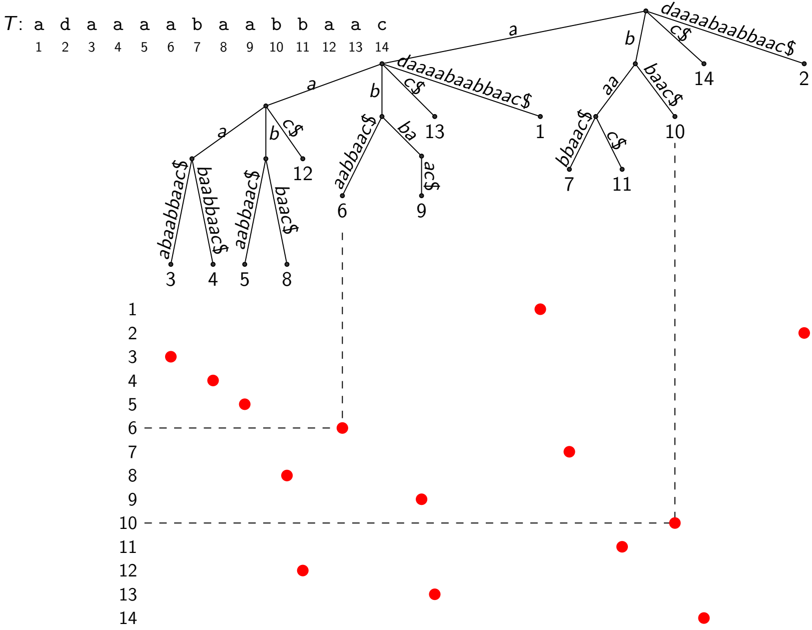

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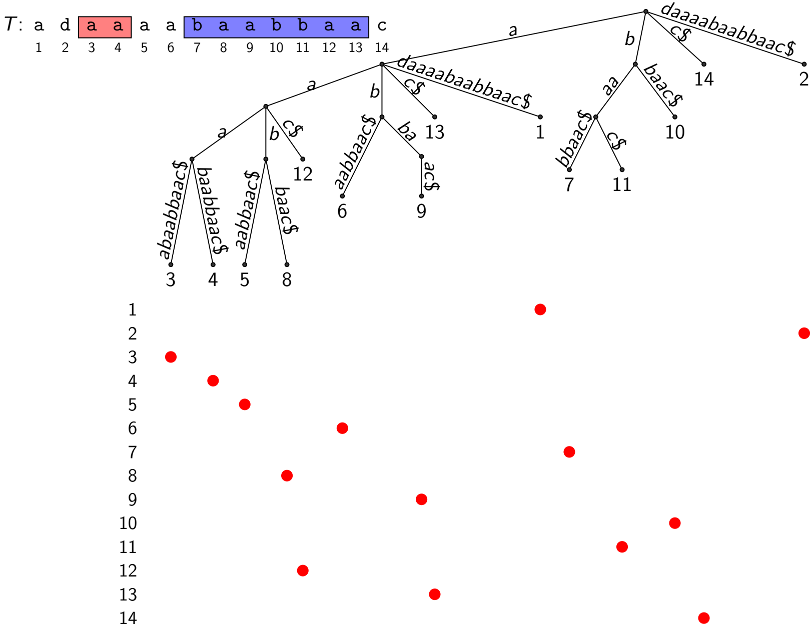

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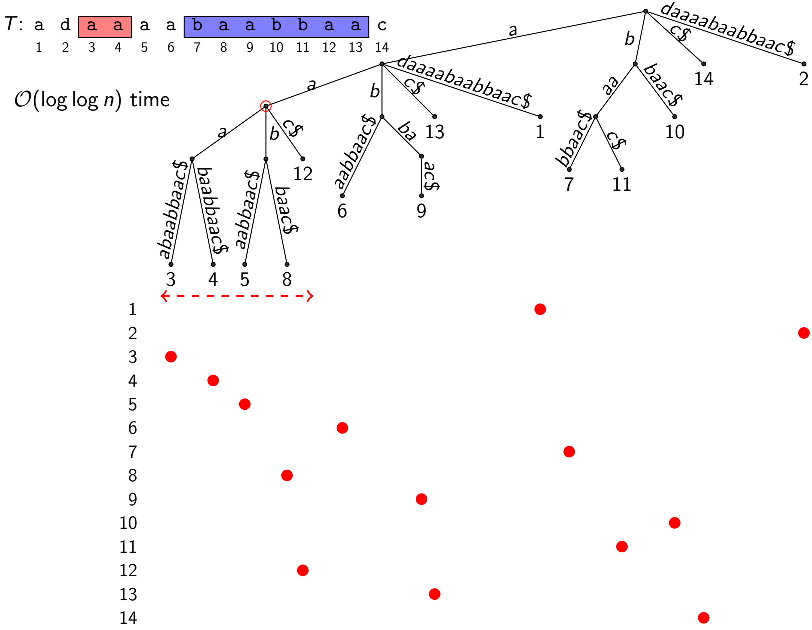

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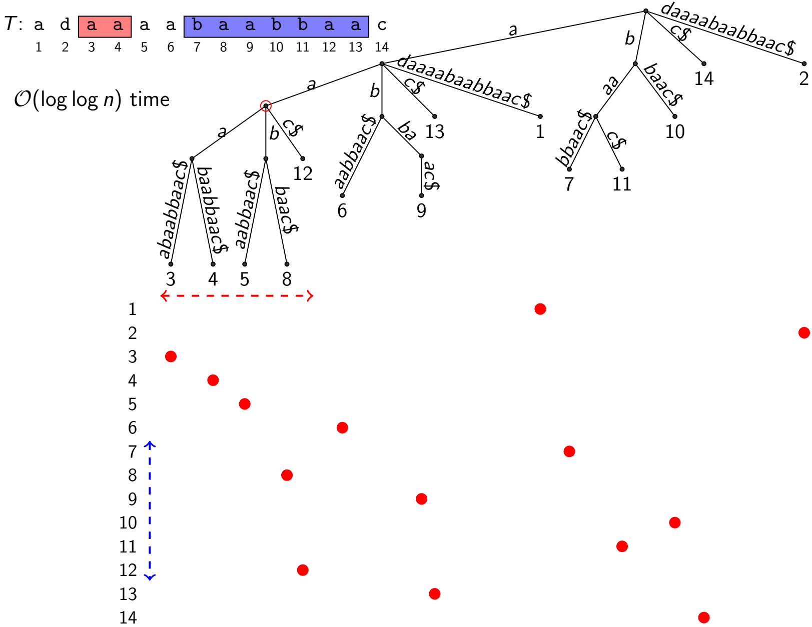

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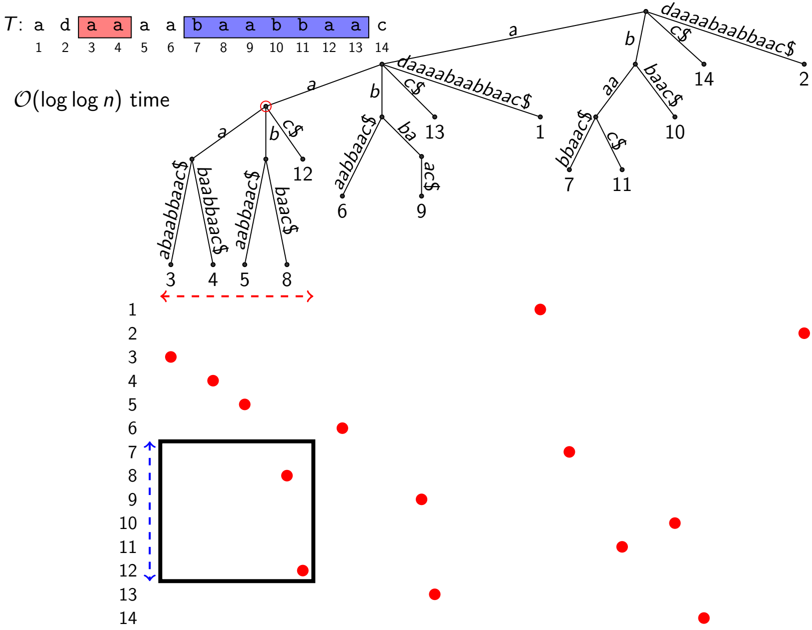

Problem Definition

Input: A text T of length n and a dictionary $\mathcal{D}$ consisting of d patterns, each given as a substring $T[\ell \mathinner{.\,.} r]$ of T.

$\mathcal{D}$:

a a a a a a

a b b a c

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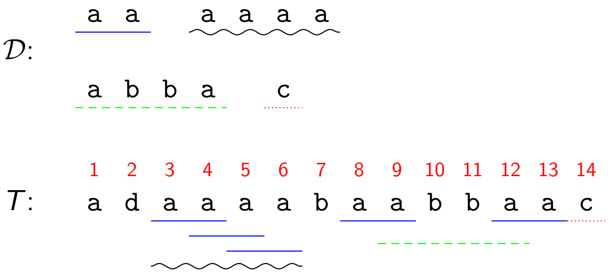

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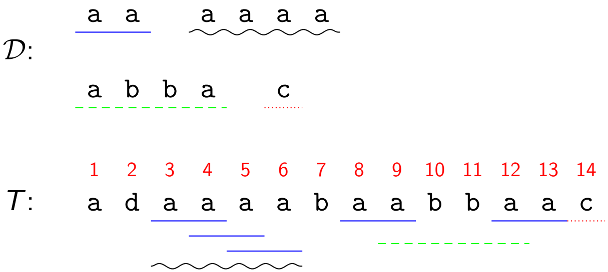

Exists(i, j)

- Decide whether at least one pattern $P \in \mathcal{D}$ occurs in $T[i \mathinner{.\,.} j]$.

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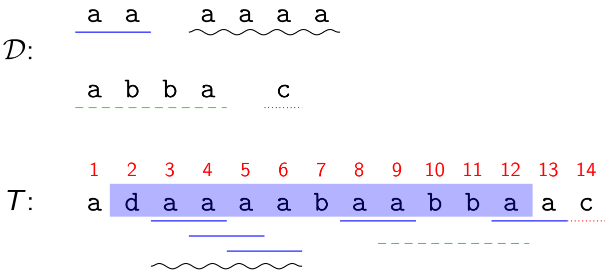

$\textsc{Exists}(i, j)$

- Decide whether at least one pattern $P \in \mathcal{D}$ occurs in $T[i \,..\, j]$.
- $\textsc{Exists}(2{,}12) = \text{true}$

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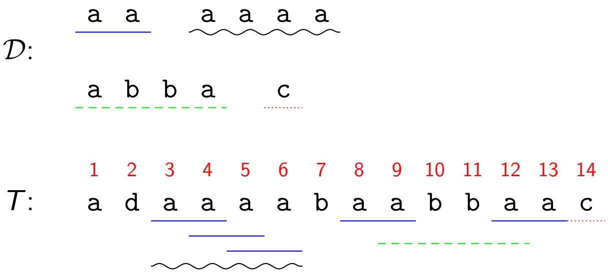

Report(i, j)

- Report all occurrences of all patterns of $\mathcal{D}$ in $T[i \,..\, j]$.

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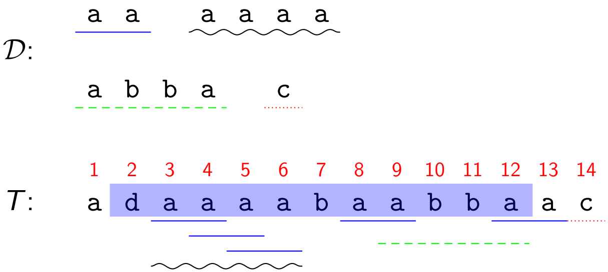

Report(i, j)

- Report all occurrences of all patterns of $\mathcal{D}$ in $T[i\,..\,j]$.
- Report(2,12) = (aa,3),(aaaa,3),(aa,4),(aa,5),(aa,8), (abba,9)

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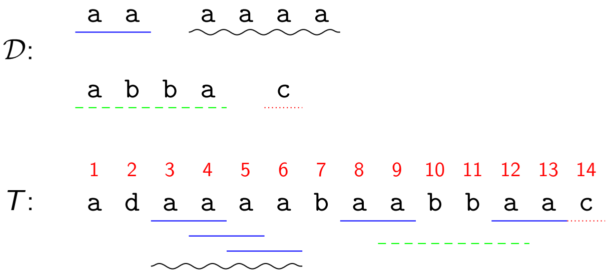
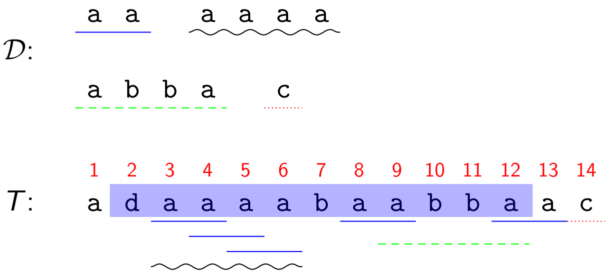


ReportDistinct(i, j)

- Report all patterns P of $\mathcal{D}$ that occur in $T[i \mathinner{.\,.} j]$.

Input: A text T of length n and a dictionary $\mathcal{D}$ consisting of d patterns, each given as a substring $T[\ell \,.\,.\, r]$ of T.



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- Report all patterns P of $\mathcal{D}$ that occur in $T[i \,.\,.\, j]$.
- ReportDistinct(2,12) $=$ `aa, aaaa, abba`

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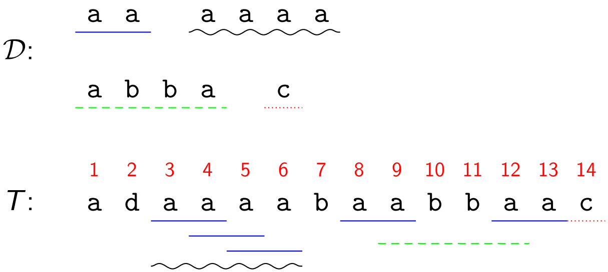

Count(i, j)

- Count the number of all occurrences of all the patterns of $\mathcal{D}$ in $T[i \mathinner{.\,.} j]$.

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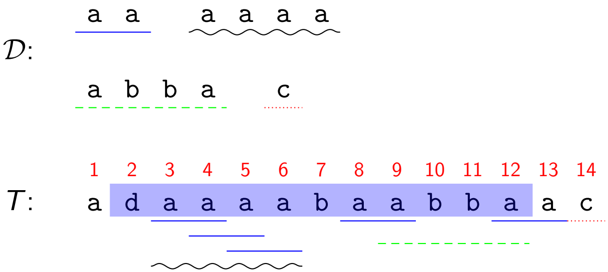

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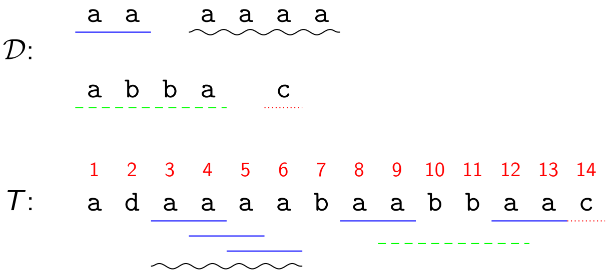

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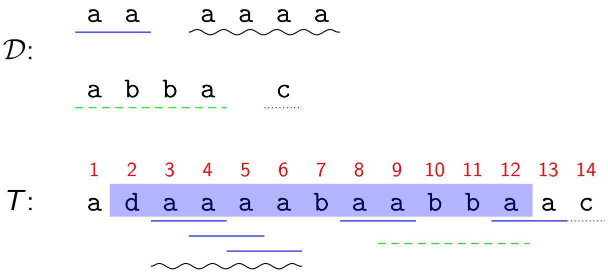

CountDistinct(i,j)

- Count all patterns of $\mathcal{D}$ that occur in $T[i \mathinner{.\,.} j]$.
- CountDistinct(2,12) $=$ 3

Our Results

Query	Preprocessing time	Space	Query time
$\textsc{Exists}(i,j)$	$\mathcal{O}(n+d)$	$\mathcal{O}(n)$	$\mathcal{O}(1)$
$\textsc{Report}(i,j)$	$\mathcal{O}(n+d)$	$\mathcal{O}(n+d)$	$\mathcal{O}(1+\lvert output\rvert)$
$\textsc{ReportDistinct}(i,j)$	$\mathcal{O}(n\log n+d)$	$\mathcal{O}(n+d)$	$\mathcal{O}(\log n+\lvert output\rvert)$
$\textsc{Count}(i,j)$	$\mathcal{O}(\frac{n\log n}{\log\log n}+d\log^{3/2} n)$	$\mathcal{O}(n+d\log n)$	$\mathcal{O}(\frac{\log^2 n}{\log\log n})$
	$\tilde{\mathcal{O}}(n+d)$	$\tilde{\mathcal{O}}(n+d)$	$\tilde{\mathcal{O}}(1+\lvert output\rvert)$

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Dynamic dictionary

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$\textsc{ReportDistinct}(i,j)$	$\mathcal{O}(n\log n+d)$	$\mathcal{O}(n+d)$	$\mathcal{O}(\log n+\lvert output\rvert)$
$\textsc{Count}(i,j)$	$\mathcal{O}(\frac{n\log n}{\log\log n}+d\log^{3/2} n)$	$\mathcal{O}(n+d\log n)$	$\mathcal{O}(\frac{\log^2 n}{\log\log n})$
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Dynamic dictionary

- Conditional lower bound: $t_{upd} \times t_{query}$ cannot be $\mathcal{O}(n^{1-\epsilon})$ for any constant $\epsilon > 0$, unless the Online Boolean Matrix-Vector Multiplication conjecture is false.

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- Conditional lower bound: $t_{upd} \times t_{query}$ cannot be $\mathcal{O}(n^{1-\epsilon})$ for any constant $\epsilon > 0$, unless the Online Boolean Matrix-Vector Multiplication conjecture is false.
- Upper bound: $t_{upd} = \tilde{\mathcal{O}}(n^{\alpha})$, $t_{query} = \tilde{\mathcal{O}}(n^{1-\alpha} + \lvert output\rvert)$ for any $0 < \alpha < 1$, for all our queries.

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$\textsc{ReportDistinct}(i, j)$

Partition $\mathcal{D}$ into $\mathcal{D}_0, \ldots, \mathcal{D}_{\lfloor \log n \rfloor}$, where

$$\mathcal{D}_k = \{P \in \mathcal{D} : 2^k \leq |P| < 2^{k+1}\} \text{ is the } k\text{-dictionary}.$$

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Assign a color col(P) to each pattern $P \in \mathcal{D}_k$.

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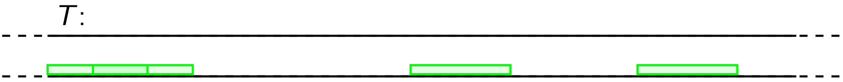

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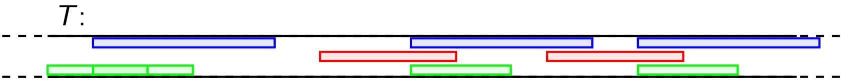

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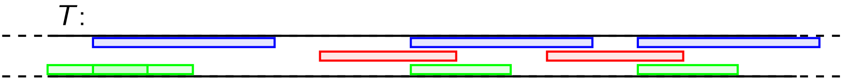

$C_k[a] = \text{col}(P_a)$, P_a the longest pattern occurring at position a

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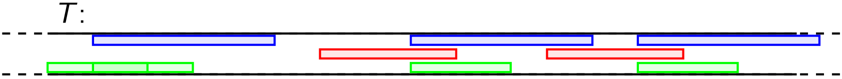

⋯	g	b	⋅⋅	⋅⋅	⋅⋅	⋅⋅	r	⋅⋅	b	⋅⋅	⋅⋅	r	⋅⋅	b	⋅⋅	⋅⋅	⋅⋅	⋯

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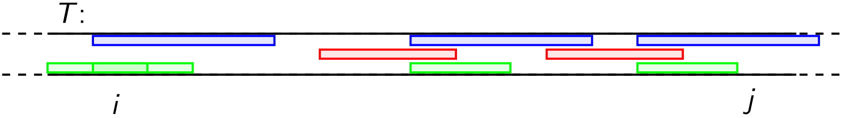

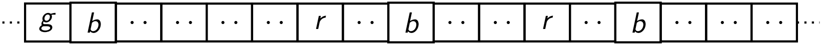

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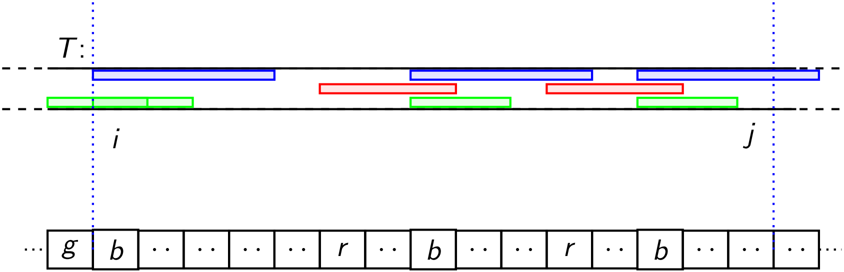

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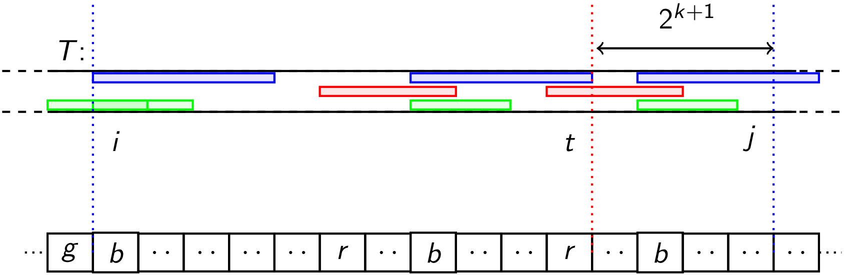

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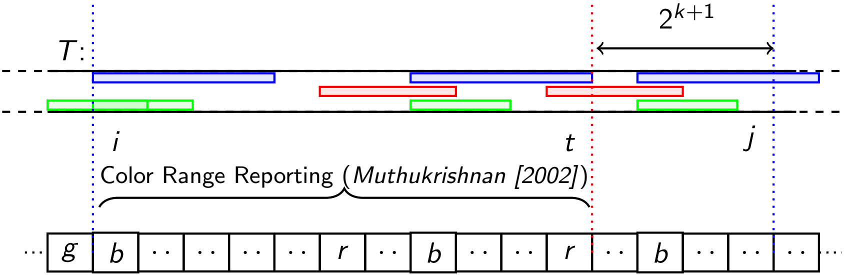

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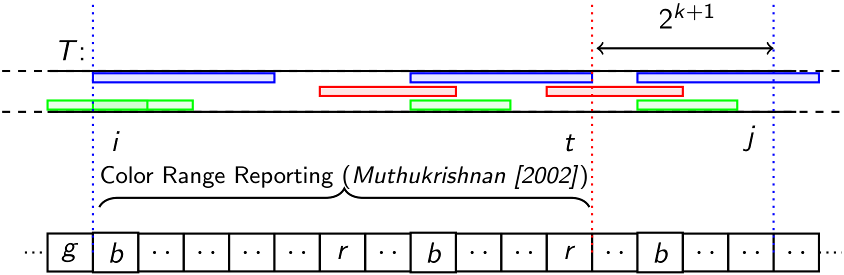

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Every $P \in \mathcal{D}_k$ that occurs in some position $a \in [i, t]$ is a prefix of one of the reported patterns.

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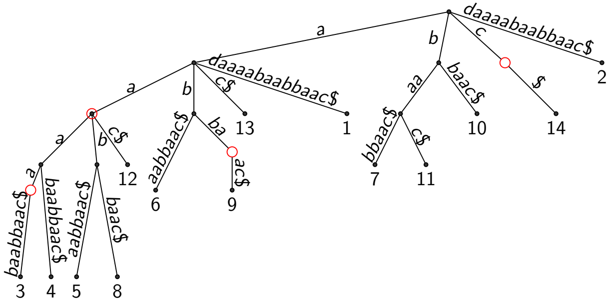

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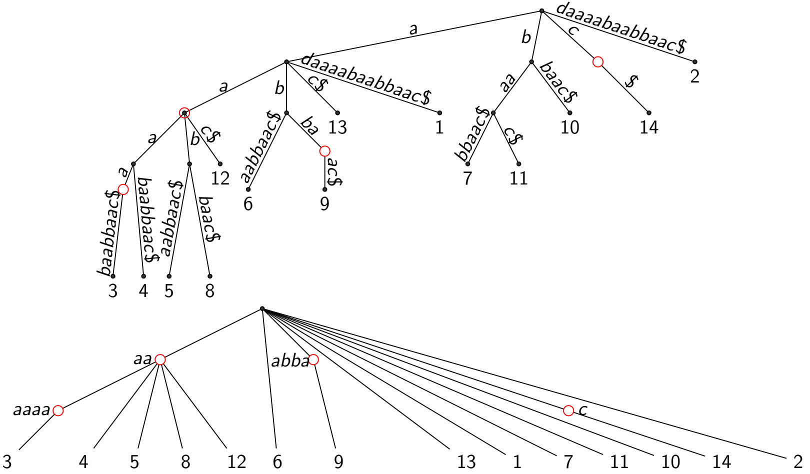

A tree representing the patterns. Prefixes in $\mathcal{D} \Leftrightarrow$ Ancestors.

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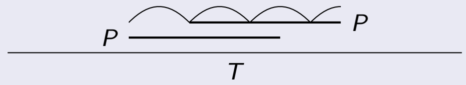

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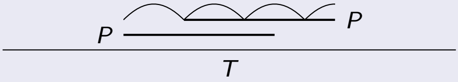

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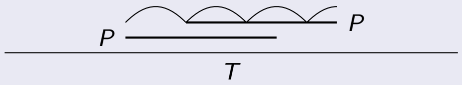

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Use $\textsc{Report}(t,j)$ for aperiodic patterns!

Each of them occurs in $T[t+1\,..\,j]$ a constant number of times.

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c	a	b	a	a	b	a	a	b	a	a	b	a	a	b	b	b	a
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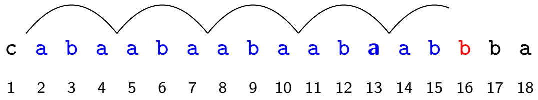

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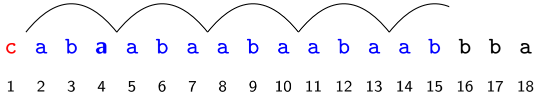

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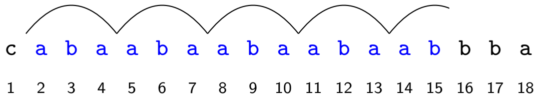

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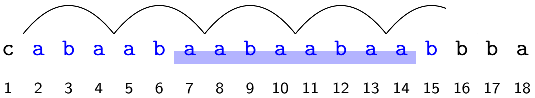

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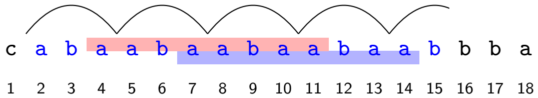

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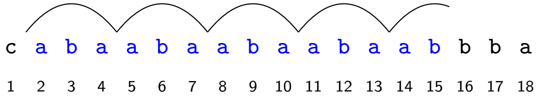

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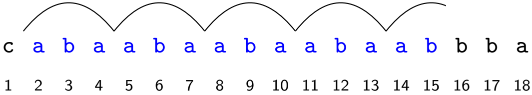

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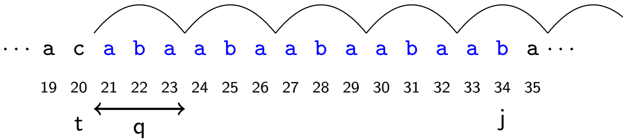

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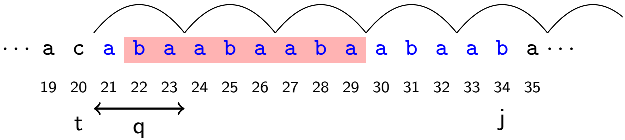

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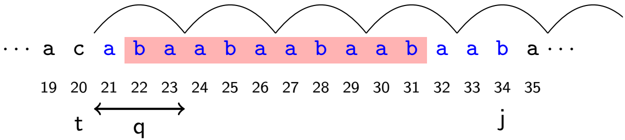

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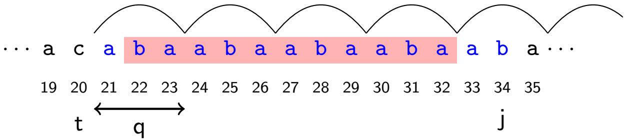

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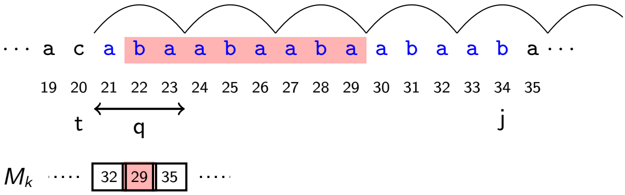

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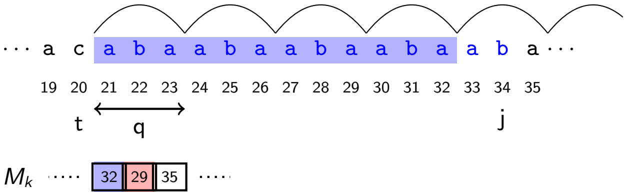

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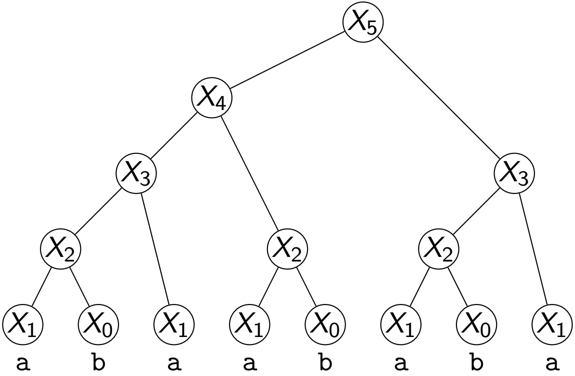

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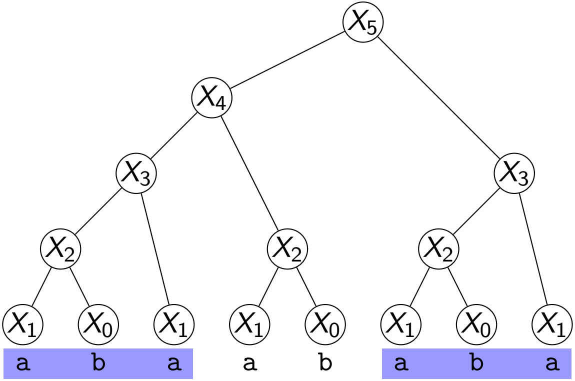

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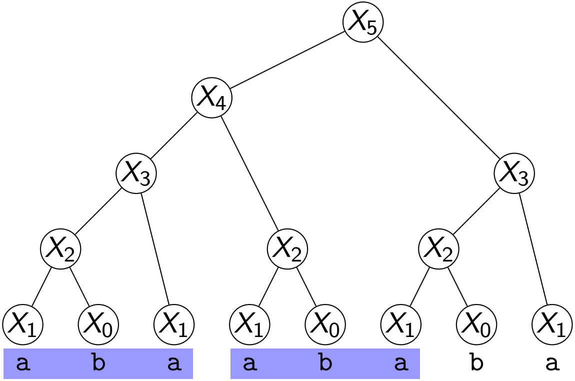

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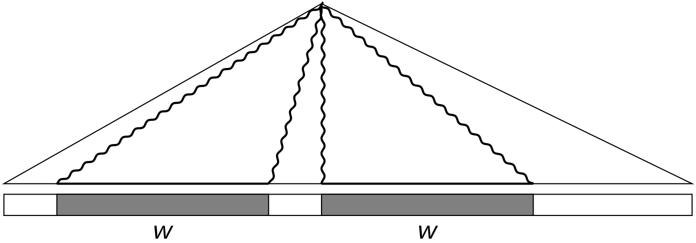

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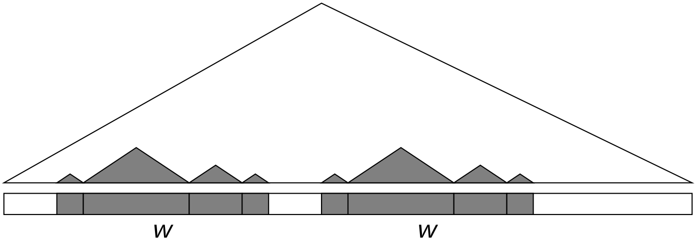

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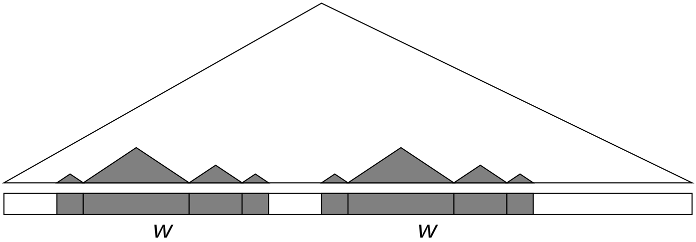

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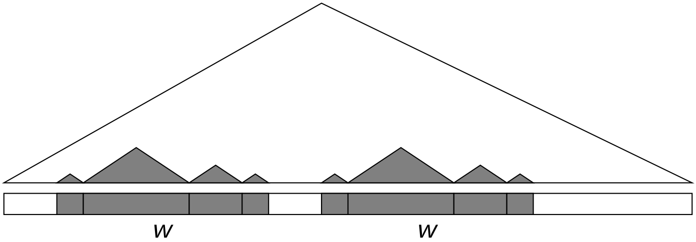

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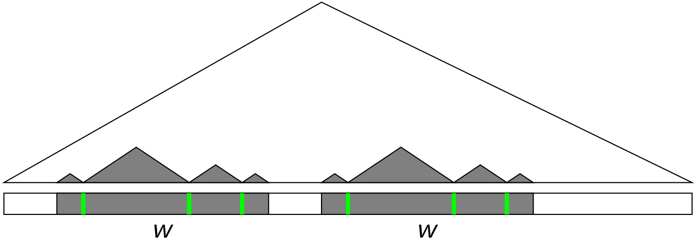

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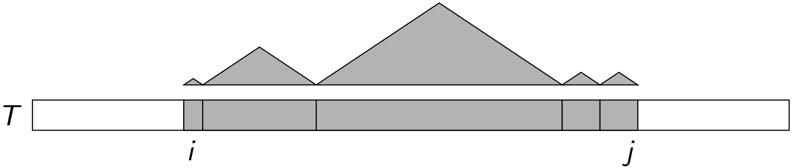

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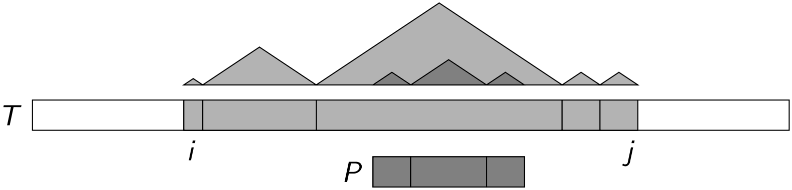

Either fully contained in a phrase – we can precompute all such counts in a bottom-up manner.

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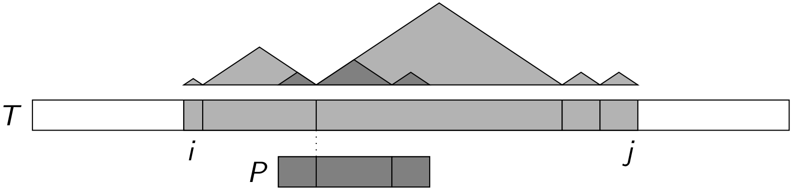

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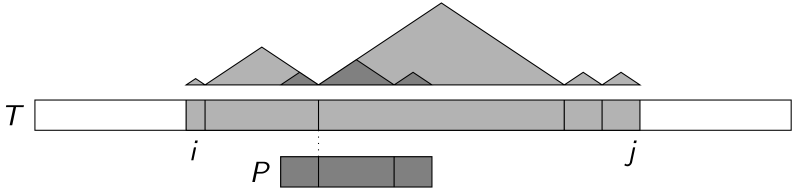

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Breakpoints-Anchor IDM

- Input: T, $\mathcal{D}$ and a set B_P of breakpoints for each $P \in \mathcal{D}$.
- Query: $\textsc{Count}_\alpha(i, j)$: the number of fragments of $T[i \,..\, j]$ that match some $P \in \mathcal{D}$ and some breakpoint from B_P is aligned with anchor α.

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Breakpoints-Anchor IDM and Wrap-up

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- Space and preprocessing time: $\tilde{\mathcal{O}}(n + \sum_{P \in \mathcal{D}} |B_P|)$
- Query time: $\mathcal{O}(\log n / \log \log n)$

$|B_P| = \mathcal{O}(\log n)$ for all $P \in \mathcal{D}$.
We need to ask $\mathcal{O}(\log n)$ such queries; one for each anchor α.

$\textsc{Count}(i, j)$

- Space and preprocessing time: $\tilde{\mathcal{O}}(n + d)$
- Query time: $\mathcal{O}(\log^2 n / \log \log n)$

Note on parsing

Simplified view of the parsing – the properties above are too good to be true. We use the parsing from the recompression technique due to Artur Jeż, and its interpretation by Tomohiro I.

Follow up work on CountDistinct(i, j):

Preprocessing time	**Space**	**Query time**	**Variant**
$\tilde{\mathcal{O}}(n + d)$	$\tilde{\mathcal{O}}(n + d)$	$\tilde{\mathcal{O}}(1)$	2-approx
$\tilde{\mathcal{O}}(n^2/m + d)$	$\tilde{\mathcal{O}}(n^2/m^2 + d)$	$\tilde{\mathcal{O}}(m)$	exact
$\tilde{\mathcal{O}}(nd/m + d)$	$\tilde{\mathcal{O}}(nd/m + d)$	$\tilde{\mathcal{O}}(m)$	exact

Table: m is an arbitrary parameter.

The End

Query	Preprocessing time	Space	Query time
$\textsc{Exists}(i,j)$	$\mathcal{O}(n+d)$	$\mathcal{O}(n)$	$\mathcal{O}(1)$
$\textsc{Report}(i,j)$	$\mathcal{O}(n+d)$	$\mathcal{O}(n+d)$	$\mathcal{O}(1+\lvert output \rvert)$
$\textsc{ReportDistinct}(i,j)$	$\mathcal{O}(n\log n+d)$	$\mathcal{O}(n+d)$	$\mathcal{O}(\log n+\lvert output \rvert)$
$\textsc{Count}(i,j)$	$\mathcal{O}(\frac{n\log n}{\log\log n}+d\log^{3/2} n)$	$\mathcal{O}(n+d\log n)$	$\mathcal{O}(\frac{\log^2 n}{\log\log n})$

Thank you! Questions?

References

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