Gaussian Interference Channel Aided by a Relay with Out-of-Band Reception and In-Band Transmission

Onur Sahin, Osvaldo Simeone, and Elza Erkip

***Abstract*—A Gaussian Interference Channel (IC) is investigated in which a relay assists two source-destination pairs. The relay is assumed to receive over dedicated orthogonal channels from the sources (e.g., over orthogonal bands or time slots, or over wired links), while it transmits in the same band as the sources. This scenario is referred to as IC assisted by an out-of-band reception/ in-band transmission relay (IC-OIR). An achievable rate region is derived for the IC-OIR that encompasses, besides the standard signal relaying, interference management via interference relaying, cancellation and precoding. The sum-capacity is found in a specific regime defined by the very strong relay-interference conditions. Numerical results validate the performance gains of interference mitigation via the relay.**

***Index Terms*—Relays, Gaussian interference channel, interference mitigation, parallel channels, power allocation, interference forwarding.**

I. Introduction

CURRENT and future generation communication systems accommodate simultaneous communication of multiple nodes that belong to the same network or possibly have distinct radio access technologies. Heterogenous networks (Het-Nets), including femtocell, picocell, and macro cell nodes as well as relays, are expected to replace the conventional infrastructure networks. However, interference becomes a major bottleneck in Het-Nets due to the difficulty of joint scheduling and coordination of nodes in a distributed manner.

The effect of relaying in interference limited systems has attracted interest recently. Relaying techniques, traditionally studied in point-to-point settings, have been investigated in the presence of multiple interfering communication links [5]-[15]. In [5]-[12], a single relay assists two interfering source-destination pairs, resulting in an interference channel (IC) with a relay. References [13]-[15] consider two hop communication where two relays assist separate information flows, modeled as a cascade of two IC's. The main conclusion of [5]-[12] is that the relay, when operating in the presence of interference, can not only perform standard *signal relaying*, but can also manage interference through: (a) *interference relaying,* where interference is boosted at the interfered destination so as to ease decoding and removal of the interfering source; (b) *interference cancellation (or neutralization),* where the relay's transmission combines destructively at the interfered destination so as to cancel the interference; and (c) *interference precoding,* where the relay performs Dirty Paper Coding (DPC) on the interference to transmit independent signal information to the intended destination.

In this paper, we study an IC aided by a relay that receives over dedicated orthogonal channels but transmits in the same band as the sources. This scenario arises, for instance, in a macro cell overlayed by closed subscriber group (CSG) femtocells. Due to the CSG set-up, the macro nodes operating in the vicinity of femto base-stations can be subject to severe interference whereas the reception of the femto nodes can also be degraded due to the high power profile of the macro base-station. A relay node, which is expected to be a fundamental component in Het-Nets, jointly helps both macro and femto base-stations in communicating to their users. While the relay transmits in the same band with the macro and femto base-stations, the backhaul links of the relay are assumed to be orthogonal such that separate time slots, frequency bands or wired links are used for the base-stations to communicate with the relay. We refer to this scenario as a Gaussian IC assisted by an out-of-band reception/ in-band transmission relay, or IC-OIR in short. Compared with cognitive relaying in [5], [7]-[9], the transmission scheme in IC-OIR depends on the source-to-relay channel capacities necessitating *message splitting* at the sources followed by *partial decode-and-forward relaying*. Message splitting provides interference reduction at the destinations by splitting the codewords and encoding them with different rates so that corresponding codewords can be decoded and cancelled at the interfered destinations [1]. On the other hand, in partial decode-and-forward relaying, after further splitting the codewords at the sources, the relay decodes only some of the codewords depending the backhaul capacity, and the rest of the codewords are transmitted directly from the sources to the destinations without relaying [4]. Also, IC-OIR model is fundamentally different from the recently investigated IC relaying models due to the relay's reception and transmission bands. Reference [6] considers in-band relay reception and transmission, [12] studies in-band relay reception and out-of-band noiseless transmission, and [11] investigates out-of-band relay reception and transmission. The relay's reception and transmission bands impact feasible transmission strategies as well as optimality of these schemes, as clearly documented in single source-destination settings [16], [17].

For the IC-OIR, we propose a general achievable rate

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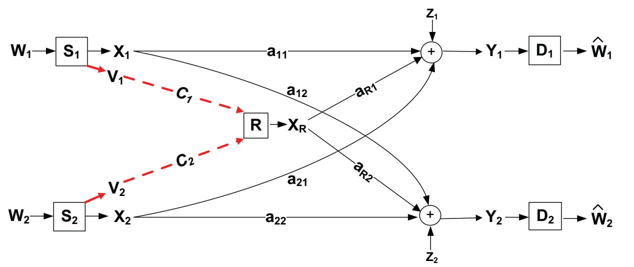

Fig. 1. Gaussian interference channel with an out-of-band reception/in-band transmission relay (IC-OIR).

region that encompasses interference relaying, cancellation and precoding. We establish *very strong relay-interference conditions*, analogous to [1], and the corresponding sum-capacity of IC-OIR, which is equal to the sum capacity of two non-interfering relay channels with sum power constraint on the relay. Numerical results demonstrate that relay interference mitigation techniques are essential in improving the data rates.

Notation: Throughout the paper we use $\mathcal{C}(x) = \log_2(1+x)$.

II. System Model

The IC-OIR model, shown in Fig. 1, is represented by

$$Y_{1,t} = a_{11}X_{1,t} + a_{21}X_{2,t} + a_{R1}X_{R,t} + Z_{1,t} \quad (1)$$

$$Y_{2,t} = a_{12}X_{1,t} + a_{22}X_{2,t} + a_{R2}X_{R,t} + Z_{2,t}, \quad (2)$$

where $X_{i,t}$ is the signal transmitted by source S_i, $X_{R,t}$ is the signal transmitted by the relay and $Y_{i,t}$ is the received signal at destination D_i at time $t = 1, ..., n$ with $i = 1, 2$. The channel coefficients a_{ij}, $i = 1, 2, R$, $j = 1, 2$ are complex valued, $\{Z_{i,t}\}$ are complex Gaussian noise processes with unit variance, independent identically distributed (i.i.d.) over both i and t. The source S_i to relay R link has capacity C_i bits/channel use, $i = 1, 2$, and it is used n times before transmission over IC takes place (out-of-band relay reception)[1]. All channel gains and source-relay link capacities are fixed and known at all terminals.

Source S_i communicates with destination D_i at rate R_i bits/channel use, $i = 1, 2$. Communication takes place over blocks of length n. Message W_i is encoded by S_i via an encoding function f_i^n: $[1, 2^{nR_i}] \to \mathbb{R}^n \times [1, 2^{nC_i}]$, which maps $W_i \in [1, 2^{nR_i}]$ into a codeword X_i^n to be sent on the IC and to a string of bits $V_i \in [1, 2^{nC_i}]$, which are sent to the relay via the dedicated S_i-R link before transmission over the IC (e.g., in the previous block). The relay, after reception of V_1, V_2, uses $f_R^n : [1, 2^{nC_1}] \times [1, 2^{nC_2}] \to \mathbb{R}^n$, to map (V_1, V_2) into a codeword $X_R^n = f_R^n(V_1, V_2)$. Codeword X_R^n is then transmitted over the IC in a synchronous manner with the sources' codewords X_i^n, $i = 1, 2$ (in-band relay transmission). We enforce power constraints $1/n \sum_{t=1}^n E[X_{i,t}^2] \leq P_i$ on the sources' codebooks, $i = 1, 2$, and $1/n \sum_{t=1}^n E[X_{R,t}^2] \leq P_R$ on the relay codebook (expectation is taken with respect to the messages). Finally, decoding at destination D_i, $i = 1, 2$, takes place using $g_i^n : \mathbb{R}^n \to [1, 2^{nR_i}]$, which maps the received signal over the IC, Y_i^n, into the estimated message $\hat{W}_i \in [1, 2^{nR_i}]$. Probability of error and achievable rates are defined in the standard manner as in [18].

Remark 1: If $C_i \to \infty$, $i = 1, 2$, the model reduces to *cognitive* relaying of [5][7][8], where the messages W_i, $i = 1, 2$, are known at the relay non-causally. This scenario will be referred to as an IC with a cognitive relay (IC-CR).

III. A General Achievable Region for IC-OIR

We propose an achievable rate region for the IC-OIR that encompasses signal relaying, interference relaying, interference cancellation and interference precoding via partial decode-and-forward transmission. Specifically, we perform message splitting at the sources with the following aims: (*i*) To reduce the effect of interference at the destinations as in the standard Han-Kobayashi scheme via splitting into "private" and "common" messages – The first are to be decoded only at the intended destination, while the second are also decoded at the interfered destination to reduce the effect of interference [2]; (*ii*) To enable signal and interference relaying/ cancellation using partial decode-and-forward [4] over parts of the private and common messages; (*iii*) To enable interference precoding, whereby the relay sends a fraction of the private messages directly to the intended destinations via DPC [3] against part of the interference.

To elaborate, source S_i partitions the bits of message W_i into independent messages as $W_i = (W_{ic'}, W_{ic''}, W_{ip'}, W_{ip''}, W_{i\widehat{p}})$ with corresponding rates $(R_{ic'}, R_{ic''}, R_{ip'}, R_{ip''}, R_{i\widehat{p}})$ such that

$$R_i = R_{ic'} + R_{ic''} + R_{ip'} + R_{ip''} + R_{i\widehat{p}}. \quad (3)$$

Specifically (i) $W_{ic'} \in [1, ..., 2^{nR_{ic'}}]$ is the common message transmitted by S_i and R jointly; (ii) $W_{ic''} \in [1, ..., 2^{nR_{ic''}}]$ is the common message transmitted by S_i only; (iii) $W_{ip'} \in [1, ..., 2^{nR_{ip'}}]$ is the private message transmitted by S_i and R jointly; (iv) $W_{ip''} \in [1, ..., 2^{nR_{ip''}}]$ is the private message transmitted by S_i only; and (v) $W_{i\widehat{p}} \in [1, ..., 2^{nR_{i\widehat{p}}}]$ is the private message transmitted from S to R via out-of-band reception links and then by R only using DPC over the signals carrying the splits $(W_{1p'}, W_{2p'})$. Notice that message splits $(W_{ic'}, W_{ip'})$ are transmitted cooperatively by source S_i and relay R, thanks to the fact that they were conveyed to the relay via the dedicated links prior to transmission. Moreover, the common splits, $(W_{ic'}, W_{ic''})$, $i = 1, 2$ are decoded at both destinations such that the overall interference at D_j, $j \neq i$ is reduced.

Proposition 1: An achievable rate region for a Gaussian IC-OIR is given by the convex hull of the set of rate pairs (R_1, R_2) with (3) and satisfying the inequalities

$$R_{1c'} + R_{1p'} + R_{1\widehat{p}} \leq C_1 \quad (4a)$$

$$R_{2c'} + R_{2p'} + R_{2\widehat{p}} \leq C_2 \quad (4b)$$

and

$$\sum_{s \in S_{1a} \cup S_{1b}} R_s \leq \mathcal{C}\left(\frac{f_s}{N_{t_1}}\right) \quad (5)$$

$$\sum_{s \in S_{2a} \cup S_{2b}} R_s \leq \mathcal{C}\left(\frac{g_s}{N_{t_2}}\right) \quad (6)$$

[1] Given the orthogonality between the source-relay links and the other links, this can be easily implemented by pipelining.

$$R_{1\widehat{p}} \leq \mathcal{C}\left(\frac{|a_{R1}|^2|\xi_{1\widehat{p}}|^2P_R}{|a_{21}|^2|\alpha_{2p''}|^2P_2+1}\right) \tag{7}$$

$$R_{2\widehat{p}} \leq \mathcal{C}\left(\frac{|a_{R2}|^2|\xi_{2\widehat{p}}|^2P_R}{|a_{12}|^2|\alpha_{1p''}|^2P_1+|a_{R2}|^2|\xi_{1\widehat{p}}|^2P_R+1}\right) \tag{8}$$

$$\text{where } f_s = \sum_{s\in S_{1a}}\left|a_{i1}\alpha_s\sqrt{P_i}+a_{R1}\xi_s\sqrt{P_R}\right|^2 + \sum_{s\in S_{1b}}|a_{i1}|^2|\alpha_s|^2P_i$$

$$g_s = \sum_{s\in S_{2a}}\left|a_{i2}\alpha_s\sqrt{P_i}+a_{R2}\xi_s\sqrt{P_R}\right|^2 + \sum_{s\in S_{2b}}|a_{i2}|^2|\alpha_s|^2P_i$$

for *all* subsets[2] $S_{1a} \subseteq \{1c', 1p', 2c'\}$, $S_{1b} \subseteq \{1c'', 1p'', 2c''\}$, $S_{2a} \subseteq \{2c', 2p', 1c'\}$ and $S_{2b} \subseteq \{2c'', 2p'', 1c''\}$, and *some* power allocation at the sources and the relay over the codewords carrying the corresponding message splits, such that $|\alpha_{ic'}|^2 + |\alpha_{ic''}|^2 + |\alpha_{ip'}|^2 + |\alpha_{ip''}|^2 \leq 1$, $i = 1, 2$, and $\sum_{s\in B}|\xi_s|^2 \leq 1$, with $B = \{1c', 2c', 1p', 2p', 1\widehat{p}, 2\widehat{p}\}$, respectively. Here $|\alpha_s|^2$ represents the proportion of power allocated at source i to the message split $s \in \{ic', ip', ic'', ip''\}$, and $|\xi_s|^2$ represents the proportion of relay power allocated to the message split $s \in B$, with all α_s and ξ_s being complex valued. We have defined

$$N_{t_1} = |a_{R1}|^2(|\xi_{1\widehat{p}}|^2+|\xi_{2\widehat{p}}|^2)P_R + |a_{21}|^2|\alpha_{2p''}|^2P_2 + |a_{21}\alpha_{2p'}\sqrt{P_2}+a_{R1}\xi_{2p'}\sqrt{P_R}|^2+1 \tag{9}$$

and similarly for N_{t2}, and used the convention in (5)-(6) that $i = 1$ for the terms corresponding to $s \in \{1c', 1p', 1c'', 1p''\}$ and $i = 2$ otherwise.

Proof: A sketch of proof is provided in Appendix A. ■

Remark 2: The rate constraints (4) impose that splits $(W_{ic'}, W_{ip'}, W_{i\widehat{p}})$ can be sent to the relay via the $S_i - R$ links. Conditions in (5) (alternatively (6)) arise as a result of the decoding constraints at D_1 (at D_2). The last two conditions (7) and (8) follow by assuming that the relay encodes $(W_{1\widehat{p}}, W_{2\widehat{p}})$ successively, by first encoding $W_{2\widehat{p}}$ using DPC against part of the remaining interference at D_2 (namely, against the codeword of $W_{1p'}$ so that the codeword $X_{2\widehat{p}}$ encoding $W_{2\widehat{p}}$ becomes dependent also on the codeword encoding $W_{1p'}$) and then $W_{1\widehat{p}}$, which is precoded via DPC against the codeword of $W_{2p'}$ as well as $W_{2\widehat{p}}$. Similarly, we can obtain a *distinct* achievable region by switching the encoding order for $(W_{1\widehat{p}}, W_{2\widehat{p}})$, i.e., by first encoding $W_{1\widehat{p}}$ whereas $W_{2\widehat{p}}$ is precoded via DPC also against $W_{1\widehat{p}}$. Then, taking the convex hull of the union of the two obtained rate regions gives us a larger achievable region.

IV. Sum-Capacity in the Very Strong Interference Regime

It is well known that in an IC, when the power gains on the interfering links are sufficiently strong, the capacity region is the same as that of the scenario where the two source-destination pairs of the IC do not interfere [1][2]. This is due to the fact that, under the so called *very strong interference* conditions, the interfering signal can be decoded first and removed without degrading the point-to-point achievable rates. Therefore, in this regime, transmitting *only common information* (and thus no private information), is optimal [1][2]. In this section, we look for analogous conditions for the IC-OIR.

[2]Including the empty set.

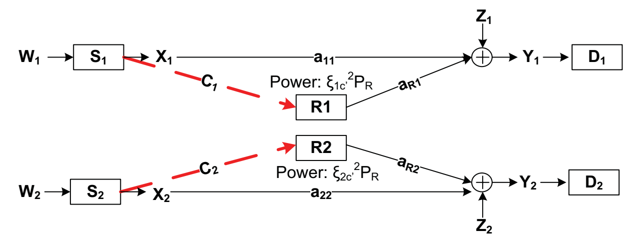

Fig. 2. Parallel relay channels with a sum relay power constraint, $\xi^2_{1c'} + \xi^2_{2c'} \leq P_R$ used in the upper bound of Prop. 2.

We first discuss the channel illustrated in Fig. 2 where we have two non-interfering (parallel) relay channels with orthogonal source-relay links (also known as relay channel with orthogonal components [4]) . The two relays $R1$ and $R2$ have a sum power constraint represented by $P_{R1} = |\xi_{1c'}|^2P_R$, $P_{R2} = |\xi_{2c'}|^2P_R$ with $|\xi_{1c'}|^2 + |\xi_{2c'}|^2 \leq 1$ (the reason for using subscript c' will be made clear below). The capacity for each of the two component relay channels follows from [4], and to find the sum capacity of the parallel relay channel in Fig. 2 we further have to optimize over all complex valued $(\xi_{1c'}, \xi_{2c'})$. This is illustrated in the next lemma.

Lemma 1: The sum-capacity of the parallel relay channel in Fig. 2 is given by

$$C_{sum} = \max_{(\alpha_{ic'},\alpha_{ic''},\xi_{ic'}),i=1,2}(R_1+R_2), \tag{10}$$

where maximization is subject to conditions

$$R_i \leq \min\left\{\begin{array}{l}\mathcal{C}\left(|a_{ii}|^2|\alpha_{ic''}|^2P_i + |a_{ii}\alpha_{ic'}\sqrt{P_i}+a_{Ri}\xi_{ic'}\sqrt{P_R}|^2\right)\\ C_i+\mathcal{C}\left(|a_{ii}|^2|\alpha_{ic''}|^2P_i\right)\end{array}\right\} \tag{11}$$

for $i = 1, 2$ with $|\alpha_{ic''}|^2+|\alpha_{ic'}|^2 \leq 1$, and $|\xi_{1c'}|^2+|\xi_{2c'}|^2 \leq 1$.

The following definition and proposition determine the channel conditions in an IC-OIR that guarantee the same sum-rate as in the non-interfering relay channels of Fig. 2.

Definition 1: An IC-OIR is in *very strong relay-interference regime* if for $i, j = 1, 2$ and $j \neq i$, we have

$$\mathcal{C}\left(\frac{|a_{ij}\alpha^{opt}_{ic'}\sqrt{P_i}+a_{Rj}\xi^{opt}_{ic'}\sqrt{P_R}|^2}{1+|a_{jj}|^2|\alpha^{opt}_{jc''}|^2P_j+|a_{jj}\alpha^{opt}_{jc'}\sqrt{P_j}+a_{Rj}\xi^{opt}_{jc'}\sqrt{P_R}|^2}\right) \geq \mathcal{C}\left(|a_{ii}\alpha^{opt}_{ic'}\sqrt{P_i}+a_{Ri}\xi^{opt}_{ic'}\sqrt{P_R}|^2\right) \tag{12a}$$

$$\mathcal{C}\left(\frac{|a_{ij}|^2|\alpha^{opt}_{ic''}|^2P_i}{1+|a_{jj}|^2|\alpha^{opt}_{jc''}|^2P_j+|a_{jj}\alpha^{opt}_{jc'}\sqrt{P_j}+a_{Rj}\xi^{opt}_{jc'}\sqrt{P_R}|^2}\right) \geq \mathcal{C}\left(|a_{ii}|^2|\alpha^{opt}_{ic''}|^2P_i\right) \tag{12b}$$

$$\mathcal{C}\left(\frac{|a_{ij}|^2|\alpha^{opt}_{ic''}|^2P_i+|a_{ij}\alpha^{opt}_{ic'}\sqrt{P_i}+a_{Rj}\xi^{opt}_{ic'}\sqrt{P_R}|^2}{1+|a_{jj}|^2|\alpha^{opt}_{jc''}|^2P_j+|a_{jj}\alpha^{opt}_{jc'}\sqrt{P_j}+a_{Rj}\xi^{opt}_{jc'}\sqrt{P_R}|^2}\right) \geq \mathcal{C}\left(|a_{ii}|^2|\alpha^{opt}_{ic''}|^2P_i+|a_{ii}\alpha^{opt}_{ic'}\sqrt{P_i}+a_{Ri}\xi^{opt}_{ic'}\sqrt{P_R}|^2\right) \tag{12c}$$

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where $\alpha_{ic'}^{opt}, \alpha_{ic''}^{opt}, \xi_{ic'}^{opt}$ are the optimal allocations that maximize C_{sum} in Lemma 1.

Proposition 2: In an IC-OIR satisfying the very strong relay-interference conditions, the sum-capacity is given by C_{sum} of Lemma 1, and is obtained by transmitting only the common message splits $(W_{ic'}, W_{ic''})$, $i = 1, 2$ in Proposition 1.

Proof: See Appendix B. ■

Remark 3: Proposition 2 states that, under the very strong relay-interference conditions, it is sufficient to perform interference relaying only (i.e., no cancellation or precoding are needed). Moreover, only common message splits are transmitted, as under the analogous very strong interference conditions for the IC [1]. Finally, it is noted that for $C_1 = C_2 = 0$ (i.e., no relay), the sum-capacity C_{sum} is maximized for $\alpha_{ic'}^{opt} = \xi_{ic'}^{opt} = 0$, $i = 1, 2$, and the conditions (12a)-(12c) reduce to the standard very strong interference conditions of the Gaussian IC [1].

V. Illustration and Discussion of the Results

In this section, we illustrate the results of Sec. III-IV through numerical examples. Throughout, we fix $P_1 = P_2 = P_R = 10$ and unit variance noise. We start with investigating the impact of the relay for a simple transmission strategy. We consider a symmetric IC-OIR with $a_{11} = a_{22} = 1e^{i\pi/4}$, $a_{R1} = a_{R2} = 1e^{i\pi/4}$, $a_{12} = a_{21} = 2e^{i\pi/4}$. Fig. 3 shows the achievable region of Proposition 1 with the sources transmitting common messages $(W_{ic''}, W_{ic'})$ only for source-to-relay capacities of $C_1 = C_2 = 0$ (i.e., no relay), $C_1 = C_2 = 0.5$ and $C_1 = C_2 \geq 4$. Note that for $C_1 = C_2 = 0$, we obtain a Gaussian IC operating under strong interference for which the considered transmission strategy is optimal [1] and thus provides the capacity region. As the source-to-relay link capacities are increased to $C_1 = C_2 = 0.5$, the relay starts helping both source-destination pairs via signal and interference relaying of the common message splits, and the rate region increases. We also observe that for $C_1 = C_2 \geq 4$, the rate region does not further improve. This is because, under these conditions, the system performance becomes equivalent to that of a IC-CR, i.e., to a system where the relay knows all the messages a priori, and thus there is no further gain to be harnessed by increasing C_1, C_2.

Following the example above, we turn to the analysis of more general strategies. We focus for simplicity on a system where C_1 and C_2 are large enough to have the same performance as an IC-CR. Fig. 4 shows the sum-rate for a symmetric IC-CR with $a_{11} = a_{22} = 1e^{i\pi/4}$, $a_{R1} = a_{R2} = 0.2e^{i\pi/4}$ versus the relative strength of interfering and direct channels defined as $\mu = |a_{21}|/|a_{22}| = |a_{12}|/|a_{11}|$. We consider different special cases of the proposed general scheme, namely transmission of "private only" messages ($W_{ip'}$, i=1,2), "common only" ($W_{ic'}$, i=1,2), and "Proposition 1", which corresponds to a fully optimized rate. We observe that the gain obtained by interference precoding via DPC is marginal, and that simple rate splitting with either only common or only private rates and interference relaying/cancellation achieve the optimized sum-rate for most of the channel gains.

To further understand the role of relaying in an interference limited system, we now consider a one-sided IC-CR. In this

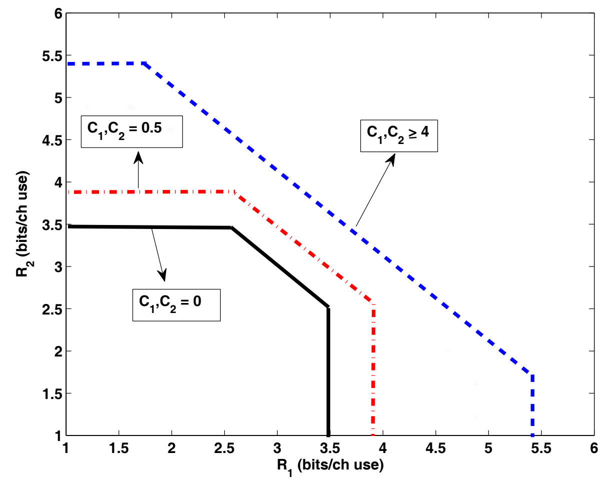

Fig. 3. Achievable region of a symmetric IC-OIR with different C_1, C_2 values. The system parameters are $P_1 = P_2 = P_R = 10$, unit noise variances, $a_{11} = a_{22} = 1e^{i\pi/4}$, $a_{R1} = a_{R2} = 1e^{i\pi/4}$, $a_{21} = a_{12} = 2e^{i\pi/4}$. The $C_1, C_2 \geq 4$ curve also corresponds to the interference channel with a cognitive relay (IC-CR) model.

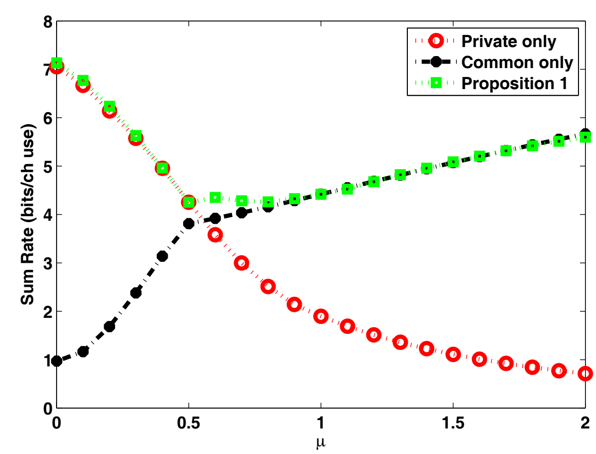

Fig. 4. Achievable sum-rate of a symmetric IC-CR for different transmission techniques. The system parameters are $P_1 = P_2 = P_R = 10$, unit noise variances, $a_{11} = a_{22} = 1e^{i\pi/4}$, $a_{R1} = a_{R2} = 0.2e^{i\pi/4}$, $\mu = |a_{21}|/|a_{22}| = |a_{12}|/|a_{11}|$.

channel, we have $a_{11} = 1e^{i\pi/4}$, $a_{22} = 4e^{i\pi/4}$, $a_{R1} = 1e^{i\pi/4}$, $a_{12} = a_{R2} = 0$, so that D_2 is not impaired by interference from S_1 but also does not receive the relay signal. In a heterogenous network, this may correspond to the case where the relay helps only the destination that is interfered, i.e., macro user which is close to the femto base-station whereas the other destination, femto user is further away from the macro base station, hence sees no interference. Fig. 5 shows again the sum-rate for various transmission techniques versus μ. In addition to "private only" and "common only", the "rate splitting" scheme illustrates achievable rates with both private and common messages, but no DPC. Notice that, since S_1 does not have link to D_2, we do not need rate splitting at this node. We recall that in the standard one-sided IC without the relay (also known as Z-channel), private information transmission is sum-rate optimal for $\mu < 1$, while transmitting common information is sum-rate optimal for $\mu \geq 1$ [19]. Here, instead, the presence of the relay complicates the optimal scheme

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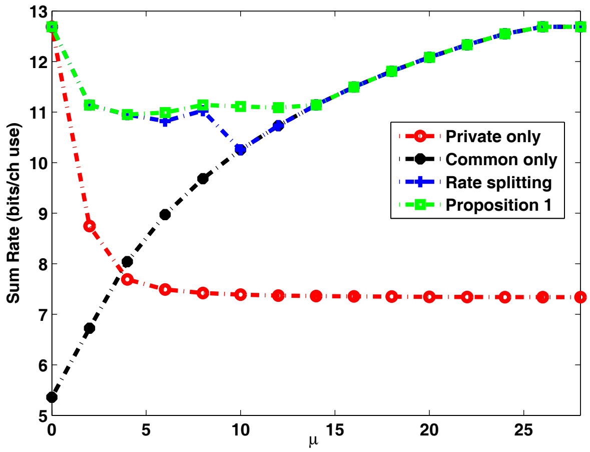

Fig. 5. Achievable sum-rate of a one-sided IC-CR for different transmission techniques. The system parameters are $P_1 = P_2 = P_R = 10$, unit noise variances, $a_{11} = 1e^{i\pi/4}$, $a_{22} = 4e^{i\pi/4}$, $a_{R1} = 1e^{i\pi/4}$, $a_{12} = a_{R2} = 0$, $\mu = |a_{21}|/|a_{22}|$.

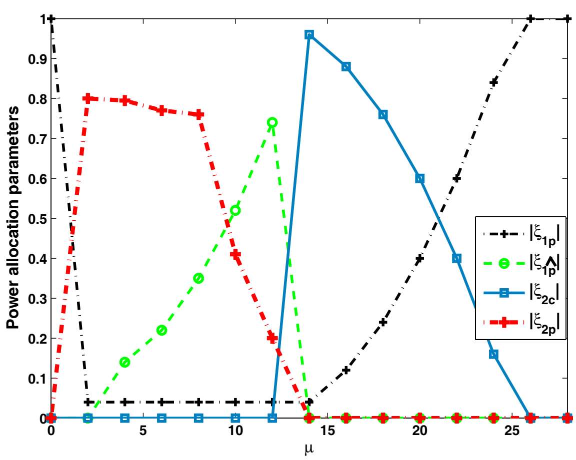

Fig. 6. Optimal relay power allocations that maximize achievable sum-rate of the one-sided IC-CR in Fig. 5 (the curve corresponding to Prop. 1). The system parameters are $P_1 = P_2 = P_R = 10$, unit noise variances, $a_{11} = a_{22} = 1e^{i\pi/4}$, $a_{R1} = 1e^{i\pi/4}$, $a_{12} = a_{R2} = 0$, $\mu = |a_{21}|/|a_{22}|$.

by requiring rate splitting for a larger range of interference levels. This is also seen in Fig. 6, which shows the optimal relay power allocations corresponding to the best achievable scheme of Prop. 1 in Fig. 5. We observe that for weak to moderate interference ratios the relay employs mostly interference precoding and interference cancellation, whereas, for larger μ, the relay switches its operation to signal and interference relaying where the latter is obtained by relaying the common information of S_2.

VI. Conclusion

Sharing relay terminals among interfering nodes provides standard relaying gains as well as performance enhancement due to interference mitigation. In this paper, we provide novel relaying schemes to exploit such gains for a Het-Net model where the relay, receiving over out-of-band links from the sources, is dedicated to help both sets of interfering links. In particular, each source judiciously performs rate splitting to minimize the effect of interference on the interfered destination while transmitting part of the information with the help of the relay (partial decode-and-forward). The relay aids the destinations by interference relaying, interference cancelation and interference precoding as well as signal relaying. We showed that under certain circumstances, the optimal relaying operation can be assessed, where the relay helps the destinations to fully decode the interference.

Appendix

A. Proof of Proposition 1

Encoding: Each source performs message splitting as discussed in Sec. III. Source i, $i = 1, 2$ randomly generates four independent codebooks using i.i.d. complex Gaussian distribution with zero mean and variance P_i, each corresponding to a message split W_s, $s \in \{ic', ic'', ip', ip''\}$. We denote the codewords in each codebook as $X_s^n(w_s)$ such that $w_s = \{1, \ldots, 2^{NR_s}\}$ are the message indices. Over n channel uses, source i transmits $X_i^n = \sum_{s\in\{ic', ic", ip', ip"\}} \alpha_s X_s^n(W_s)$ where α_s are defined in Proposition 1. Moreover, source i transmits messages $(W_{ic'}, W_{ip'}, W_{i\widehat{p}})$ to the relay via the finite capacity links, using $V_i = (W_{ic'}, W_{ip'}, W_{i\widehat{p}})$, $i = 1, 2$. Conditions (4) ensure correct reception at the relay. Upon reception of (V_1, V_2), the relay transmits $X_R^n(V_1, V_2) = X_{Rc'}^n(W_{1c'}, W_{2c'}) + X_{Rp'}^n(W_{1p'}, W_{2p'}) + X_{R\widehat{p}}^n(W_{1\widehat{p}}, W_{2\widehat{p}})$, where $X_{Rc'}^n = \xi_{1c'}\sqrt{\frac{P_R}{P_1}}X_{1c'}^n + \xi_{2c'}\sqrt{\frac{P_R}{P_2}}X_{2c'}^n$, $X_{Rp'}^n = \xi_{1p'}\sqrt{\frac{P_R}{P_1}}X_{1p'}^n + \xi_{2p'}\sqrt{\frac{P_R}{P_2}}X_{2p'}^n$ and $X_{R\widehat{p}}^n = \xi_{1\widehat{p}}\sqrt{\frac{P_R}{P_1}}X_{1\widehat{p}}^n + \xi_{2\widehat{p}}\sqrt{\frac{P_R}{P_2}}X_{2\widehat{p}}^n$. The codewords $X_{i\widehat{p}}^n$ are generated using interference precoding in the following way. Encoding for $W_{2\widehat{p}}$ via $X_{2\widehat{p}}^n$ is performed first, by using DPC[3] over the interference signal $\left(a_{12}\alpha_{1p'} + a_{R2}\xi_{1p'}\sqrt{\frac{P_R}{P_1}}\right)X_{1p'}^n$. Encoding for $W_{1\widehat{p}}$ via $X_{1\widehat{p}}^n$ is performed next by DPC over $\left(a_{21}\alpha_{2p'} + a_{R1}\xi_{2p'}\sqrt{\frac{P_R}{P_2}}\right)X_{2p'}^n + a_{R1}\xi_{2p'}X_{2\widehat{p}}^n$, which contains both codewords $X_{2p'}^n$ and $X_{2\widehat{p}}^n$ (see Remark 2).

Decoding: Destination 1 first jointly decodes the splits $(W_{1c'}, W_{1c''}, W_{2c''}, W_{2c'}, W_{1p''}, W_{1p'})$ via a standard joint typicality decoder, by treating the signals $X_{i\widehat{p}}^n(W_{i\widehat{p}})$, $i = 1, 2$ and $X_{2p''}^n(W_{2p''})$, $X_{2p'}^n(W_{2p'})$ as noise leading to constraints (5)-(6) in Proposition 1 (similarly for D_2, see [18, Part-II Lecture Notes 4] for a discussion of how the constraints in eqn (5)-(6) arise and the error-event analysis). The corresponding codewords are then subtracted from the received signal. Decoding of $W_{i\widehat{p}}$, at D_i, $i = 1, 2$ is finally performed following standard DPC decoding as given in [3, Sec. II], where D_2 treats $X_{1\widehat{p}}^n$ as noise leading to (7)-(8).

B. Proof of Proposition 2

The sum capacity in Lemma 1 also gives an upper bound for the sum-rate in the IC-OIR. This can be proved by using a genie that gives W_j to destination i, $j \neq i$.[4]

[3] Message W is coded via DPC over interference S^n by transmitting a sequence $U^n(W, S^n)$ which is obtained by finding in the subcodebook of codewords U^n mapped to message W one codeword that is jointly typical (i.e., "matches") the sequence S^n. Details can be found in [3, Sec. II] and [18, Part II-Lecture Notes 7].

[4] Using Fano inequality and defining the correlations among the source and relay inputs, the result follows from standard arguments and the (conditional) entropy maximization theorem [18, Part I-Lecture Notes 2].

For achievability, we consider a special case of the achievable region in Proposition 1, in which only common messages $(W_{ic'}, W_{ic''})$ are sent. Moreover, we focus on a simplified decoding rule in which destination D_i first decodes messages $(W_{jc'}, W_{jc''})$, $j \neq i$, from the interfering source, performs cancellation of the decoded codewords and finally decodes the intended messages $(W_{ic'}, W_{ic''})$. This leads to the rate region defined by $R_{1c'} \leq C_1$ and

$$R_{1c'} \quad \leq \quad \min\left\{ \begin{array}{c} \mathcal{C}\left(|a_{11}\alpha_{1c'}\sqrt{P_1} + a_{R1}\xi_{1c'}\sqrt{P_R}|^2\right), \\ \mathcal{C}\left(\frac{|a_{12}\alpha_{1c'}\sqrt{P_1}+a_{R2}\xi_{1c'}\sqrt{P_R}|^2}{den_1}\right) \end{array} \right\} \tag{13}$$

$$R_{1c''} \quad \leq \quad \min\left\{ \begin{array}{c} \mathcal{C}\left(|a_{11}|^2|\alpha_{1c''}|^2 P_1\right), \\ \mathcal{C}\left(\frac{|a_{12}|^2|\alpha_{1c''}|^2 P_1}{den_1}\right) \end{array} \right\} \tag{14}$$

and

$$\begin{aligned} R_1 &= R_{1c'} + R_{1c''} \\ &\leq \min\left\{ \begin{array}{l} \mathcal{C}\left(num_1\right), \\ \mathcal{C}\left(\frac{|a_{12}|^2|\alpha_{1c''}|^2 P_1 + |a_{12}\alpha_{1c'}\sqrt{P_1}+a_{R2}\xi_{1c'}\sqrt{P_R}|^2}{den_1}\right) \end{array} \right\}, \end{aligned} \tag{15}$$

where

$$num_1 = |a_{11}|^2|\alpha_{1c''}|^2 P_1 + |a_{11}\alpha_{1c'}\sqrt{P_1} + a_{R1}\xi_{1c'}\sqrt{P_R}|^2$$
$$den_1 = 1 + |a_{22}|^2|\alpha_{2c''}|^2 P_2 + |a_{22}\alpha_{2c'}\sqrt{P_2} + a_{R2}\xi_{2c'}\sqrt{P_R}|^2$$

and similarly for S_2 rates by simply switching the indices $1 \to 2$ and $2 \to 1$. Parameters α_s and ξ_s are defined as in Proposition 1. Now, imposing that the first terms in the $\min\{\}$ of (13)-(15) are smaller than the second, and the analogous inequalities for S_2, we obtain conditions, dependent on the power allocation parameters $\alpha_{ic'}, \alpha_{ic''}, \xi_{ic'}$, $i = 1, 2$, under which the achievable region reduces to (IV). Now, if such conditions are true for the values $(\alpha_{ic'}^{opt}, \alpha_{ic''}^{opt}, \xi_{ic'}^{opt})$ $i = 1, 2$ that maximize C_{sum} in Lemma 1, we can conclude that the considered scheme is optimal, in that it achieves the sum-capacity C_{sum} of the ideal system reported in Lemma 1. These conditions are given by the very strong relay-interference conditions (12a)-(12c).

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