Random signl analysis I (ECE673)
Assignment 9

The solution for this assignment will be posted on Nov. 22th (in preparation for the quiz).

1) Consider the random process defined as

$$X[n] = 2U[n] - 4U[n-1],$$

where $U[n]$ is a white noise with zero mean and variance $\sigma^2 = 1$.
(i) Is this process WSS? If so, evalutate, auto-correlation sequence (see previous assignment) and power spectral density.
(ii) Generate a realization of 1000 samples of $X[n]$ by using MATLAB (see previous assignment). Based on this realization, estimate the power spectral density and plot the estimate. Compare the estimate with the true power spectral density.
(iii) Can you propose a method to improve the estimate at the previous point? Verify by using MATLAB that the proposed technique improves the performance by plotting the corresponding estimate.

2) Let us consider the problem of prediction for the random process studied at the previous point. In particular, we would like to obtain the optimal linear estimate of $X[n+k]$ given the observation $X[n]$:

$$\hat{X}[n+k] = aX[n] + b.$$

(i) Consider at first the prediction at one step, i.e., $k = 1$. Find the correlation coefficient between $X[n]$ and $X[n+1]$. Based on this calculation, do you expect linear prediction to be effective? Evaluate the optimal predictor $\hat{X}[n+1]$ and the corresponding mean square error.
(ii) Let us now set $k = 2$. Find the correlation coefficient between $X[n]$ and $X[n+2]$. Based on this calculation, do you expect linear prediction to be effective? Evaluate the optimal predictor $\hat{X}[n+2]$ and the corresponding mean square error.
(iii) How would you generalize the results at point (ii) for $k > 2$?