Random signl analysis I (ECE673)
Assignment 7

The due date for this assignment is Wednesday Nov. 8th

1) (i) Given the independent random variables $X_1 \sim Ber(0.3)$ and $X_2 \sim Ber(0.6)$, evaluate the mean vector and covariance matrix of the transformed random vector

$$\mathbf{Y} = \begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix} = \begin{bmatrix} X_1 + X_2 \\ X_2 \end{bmatrix}.$$

(ii) Write a MATLAB program that estimates mean vector and covariance matrix of $\mathbf{Y}$ from Monte Carlo simulations. Compare the simulation results with the analysis performed at point (i). Please include the MATLAB code and numerical outcome.
2) Two lightbulbs have times to failure described by random variables X and Y (measured in months) respectively with joint PDF

$$p_{X,Y}(x,y) = 10^{-4} \exp[-0.01(x+y)]u(x)u(y),$$

where it is recalled that $u(x)$ is the step function ($u(x) = 1$ for $x \geq 0$ and $u(x) = 0$ for $x < 0$). (i) Evaluate the marginal PDFs of X and Y. Are they independent? (ii) Calculate the mean vector $E[\mathbf{Z}]$ with $\mathbf{Z} = \begin{bmatrix} X \\ Y \end{bmatrix}$ and the covariance matrix $\mathbf{C_Z}$. (iii) Evaluate the probability that both lightbulbs fail before 50 months.
3) The amplitudes of two voice signals (X_1 and X_2) are modelled as bivariate Gaussian variables (measured in Volt) with zero mean , variance 1 and correlation coefficient $\rho = 0.8$. (i) Plot the joint PDF using MATLAB. In particular, show both the tri-dimensional plot (using the command *mesh*) and the countour lines (using the command *contour*). (ii) Evaluate the probability that the amplitude of the second signal is larger than the first by 1 Volt ($P[X_2 - X_1 > 1]$).
4) The voice signals defined in the previous point are passed through an amplifier that accepts a maximum signal level of 2.5 (i.e., the dynamic range of the amplifier is $[-2.5, 2.5]$). Knowing that $X_2 = 2$, what is the probability that X_1 is outside the dynamic range of the amplifier?