Random signal analysis I (ECE673)
Assignment 3

The due date for this assignment is Wednesday Sept. 27th
Please provide detailed answers.

1. (Problem 4.41) A two-state Markov chain modeling dependent Bernoulli trials has state transition probabilities $P[0|0] = 1/4$, $P[0|1] = 3/4$ and the initial state probability of $P[0] = 1/2$. Draw the Markov state diagram. What is the probability of the sequence (tuple) $(0, 1, 0, 1, 0)$?

2. At a party a large barrel is filled with 99 gag gifts and 1 diamond ring, all enclosed in identical boxes. Each person at the party (say that there are 10 people) is given a chance to pick a box from the barrel, open the box to see if the diamond is inside, and, if not, to close the box and return it to the barrel. (*i*) Define the probabilistic model (sample space, probability assignment). (*ii*) Define a random variable that measures the number of lucky persons that draw the diamond ring. What is its probability mass function? (*iii*) Evaluate the probability that only one person finds the diamond ring. (*iv*) If the number of attendees and the number of gag gifts were very large, how could we approximate the probability mass function of the number of persons choosing the diamond ring?

3. This exercise is meant to illustrate once again the relationship between analysis and computer simulations.

   (*i*) At first, write down the probability mass function (PMF) $p_X[k]$ of $X \sim bin(2, 0.5)$.

   (*ii*) Then, write a MATLAB code that generates the random variable.X.

   (*iii*) Modify your code so as to obtain an estimate of $p_X[k]$ through Monte Carlo iterations. In particualr, estimate $p_X[k]$ by evaluating the number of Monte Carlo iterations that yield any possible value in the range $\mathcal{S}_X$ of X (i.e, evaluate the histogram of X). Compare your result with point (*i*). Increasing the number of realizations (Monte Carlo simulations) improves the estimate?

   Please include your code and numerical outcomes.