Random signal analysis I (ECE673)
Solution assignment 2

1. What is the probability of having only females in a class of N students?

   Solution: The sample space reads

$$\mathcal{S} = \{(z_1, ..., z_N) : z_i \in \{M, F\}\},$$

   that contains $N_{\mathcal{S}} = 2^N$ simple events. The event "class with only females" contains only the simple event $(F, F, ..., F)$, therefore the required probability is

$$P[(F, F, ..., F)] = \frac{1}{2^N}.$$

2. (Problem 4.9) Provide a counterexample to show that the statement $P[A|B]+P[A|B^c] = 1$ is false.

   Solution: With the definitions in the figure below

$$P[A] = P[A|B] = P[A|B^c] = 1/4$$

3. (Problem 4.13) A digital communication system transmits one of the three values $-1, 0, 1$. Due to impairments on the channel, the receiver sometimes makes an error. The error rates are 12.5% if -1 is transmitted, 75% if 0 is transmitted and 12.5% if 1 is transmitted. If the probabilities for the various symbols being transmitted are $P[-1] = P[1] = 1/4$ and $P[0] = 1/2$, find the probability of error. Repeat the problem with $P[-1] = P[1] = P[0]$ and explain your results.

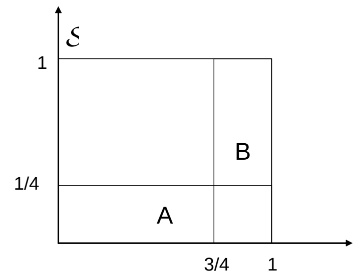
   Solution: Define the error event as E. Moreover, define as $P[i]$ the probability of the event "i has been transmitted". For the law of total probability, the probability of error reads

$$\begin{aligned} P[E] &= P[E|-1]P[-1] + P[E|0]P[0] + P[E|1]P[1] = \\ &= \frac{1}{8}\frac{1}{4} + \frac{3}{4}\frac{1}{2} + \frac{1}{8}\frac{1}{4} = \frac{7}{16}. \end{aligned}$$

   On the other hand, if the three symbols are equally likely, we have

$$\begin{aligned} P[E] &= P[E|-1]P[-1] + P[E|0]P[0] + P[E|1]P[1] = \\ &= \frac{1}{8}\frac{1}{3} + \frac{3}{4}\frac{1}{3} + \frac{1}{8}\frac{1}{3} = \frac{1}{3}. \end{aligned}$$

   The probability of error in the first case is larger than in the second since the symbol with largest conditional probability of error (0) is more likely to be transmitted in the first case.



4. (Problem 4.15) A sample space is given by $\mathcal{S} = \{(x, y) : 0 \leq x \leq 1, 0 \leq y \leq 1\}$. Determine $P[A|B]$ for the events

$$\begin{aligned} A &= \{(x, y) : y \leq 2x, 0 \leq x \leq 1/2 \text{ and } y \leq 2 - 2x, 1/2 \leq x \leq 1\} \\ B &= \{(x, y) : 1/2 \leq x \leq 1, 0 \leq y \leq 1\} \end{aligned}$$

Are events A and B independent?

Solution: The marginal probabilities read

$$\begin{aligned} P[A] &= 1/2 \\ P[B] &= 1/2, \end{aligned}$$

the joint probability is

$$P[AB] = 1/4$$

and the conditional probabilities are

$$\begin{aligned} P[A|B] &= P[AB]/P[B] = 1/2 = P[A] \qquad (1) \\ P[B|A] &= P[AB]/P[A] = 1/2 = P[B]. \qquad (2) \end{aligned}$$

Either one of conditions (1) and (2) is enough to conclude that the events A and B are independent.