Energy-Neutral Wireless Sensor Networks

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Introduction



The New York Times

The Internet Gets Physical December 17, 2011

Low-cost sensors, clever software and advancing computer firepower are opening the door to new uses in energy conservation, transportation, health care and food distribution.

... The concept has been around for years, sometimes called the Internet of Things or the Industrial Internet.

... "the sensor-aware planetary computer."

Introduction

 Energy-harvesting enables new applications for sensor networks (e.g., monitoring flocks of animals, structural health monitoring, ...)



• Energy neutrality requires to balance the use of available and predicted energy

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• Wireless sensor networks have source-channel coding capabilities



• The energy spent for *source digitization* is comparable that used for communication [Lu et al 03] [He and Wu 06] [Barr Asanovic 06]



[He and Wu 06]

Introduction

• The energy spent for *source coding* is comparable to that used for channel coding [Lu et al 03] [He and Wu 06]





Introduction

• Ex.: WLAN card with web data [Barr Asanovic 06]



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• Energy-neutral wireless sensor networks with source digitization and transmission costs



1. Point-to-point link:

- Optimal energy management policies (long-term average distortion) with arbitrarily large battery and data queue

- Trade-off distortion vs. data queue and battery sizes

Overview



• 2. Multi-hop network:

- Optimal joint routing and energy management policies (long-term average distortion) with arbitrarily large battery and data queue

- Trade-off distortion vs. data queue and battery sizes

- Energy-harvesting, transmission energy only:
 - Information theory [Sharma et al 10][Ozel and Ulukus 10][Ozel et al 10][Devillers and Gunduz 11]
 - Medium Access Control [lannello et al 10][Jeon and Ephremides 11]
 - Routing and queuing [Kansal et al 06][Lin et al 07][Gatzianas et al 10][Huang and Neely 10]
- Battery-Powered, source digitization and transmission energy: [Neely and Sharma 08][Mastronarde and van der Schaar 10]

Point-to-Point Link: System Model



- M samples of a given source measured in each slot
- Source state $H_{s,k} \in \mathcal{H}_{s}$ stationary ergodic process $\sim p_{\mathcal{H}_{s}}\left(h_{s}
 ight)$
- Bits/source symbol generated by the source encoder: f(D_k, E_{s,k}, H_{s,k}) (separately continuous convex and non-increasing in D_k and E_{s,k})



- Channel state: $H_{t,k} \in \mathcal{H}_t$ stationary ergodic $\sim p_{H_t}(h_t)$
- N channel uses (b = N/M bandwidth ratio)
- Bits per channel use delivered successfully to the destination: $g(H_{t,k}, E_{t,k})$ (continuous, concave and non-decreasing in $E_{t,k}$)
- Data queue evolution (*infinite size*):

$$X_{k+1} = [X_k - g(H_{t,k}, E_{t,k})]^+ + f(\bigcup_{a} k, E_{s,k}, H_{s,k})_{\frac{1}{a}}$$



- Energy arrival *E_{h,k}* stationary ergodic process ~ *p_{E_h}*(*e*)
 [joule/channel use]
- Energy Storage System (ESS) of *infinite size*:

$$E_{k+1} = [E_k - (E_{s,k} + E_{t,k})]^+ + E_{h,k+1}$$

Definition of Policy



• Policy of Energy management unit (EMU): $\pi := \{\pi_k\}_{k \ge 1}, \text{ where } \pi_k := \{D_k(S^k), E_{s,k}(S^k), E_{t,k}(S^k)\}$ dependent on past system states $S^k = \{S_1, \dots, S_k\}$, where $S_i = \{E_i, X_i, H_{s,i}, H_{t,i}\}$

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• Minimize over π

$$ar{D} = \limsup_{n o \infty} rac{1}{n} \sum_{k=1}^n \mathbb{E}[D_k]$$

subject to data queue is stability

 Data queue (steady state) stability: Distribution of X_k is asymptotically stationary and proper (so that Pr (X_k = ∞) → 0)

Theorem

The minimum distortion \bar{D} under stability constraints is lower bounded by the solution of the convex problem

$$\begin{split} \bar{D} &\geq \bar{D}^* = \arg\min\sum_{h_s} p_{H_s}(h_s) D^{h_s} \\ &\sum_{h_s} p_{H_s}(h_s) f^{h_s} \left(D^{h_s}, E_s^{h_s} \right) \leq \sum_{h_t} p_{H_t}(h_t) g^{h_t} \left(E_t^{h_t} \right), \\ s.t. \quad \sum_{h_s} p_{H_s}(h_s) E_s^{h_s} \leq (1-\alpha) \mathbb{E} \left[E_{h,k} \right], \\ &\sum_{h_t} p_{H_t}(h_t) E_t^{h_t} \leq \alpha \mathbb{E} \left[E_{h,k} \right], \end{split}$$

where minimization is done over the parameters $D^{h_s} \ge 0$, $E_s^{h_s} \ge 0$ for $h_s \in \mathcal{H}_s$, $E_t^{h_t} \ge 0$ for $h_t \in \mathcal{H}_t$, and $0 < \alpha < 1$.

 $\textit{Proof}\colon$ Based on G/G/1 queuing results and on identifying appropriate variables.

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Theorem

A policy π that achieves a distortion arbitrarily close to optimal is given by

$$\begin{cases} D_k = D^{h_s} & \text{for } H_{s,k} = h_s \\ E_{s,k} = \min \left[(1 - \alpha) E_k, E_s^{h_s} \right] & \text{for } H_{s,k} = h_s \\ E_{t,k} = \min \left[\alpha E_k, E_t^{h_t} \right] & \text{for } H_{t,k} = h_t \end{cases}$$

where parameters D^{h_s} , $E_s^{h_s}$, $E_t^{h_t}$ and $0 < \alpha < 1$ are obtained by solving the problem in the Theorem above with the three constraints modified by subtracting a parameter $\epsilon > 0$ arbitrarily small to the right-hand sides.

- Stationary and separable policy:
- Source coding parameters $E_s^{h_s}$, D^{h_s} adapted only to the source quality $H_{s,k} = h_s$
- Channel coding parameter $E_t^{h_t}$ adapted only to the fading state $H_{t,k} = h_t$

Optimal Policy: Comments



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Optimal Policy: Comments





- Average energy used $\leq \mathbb{E}\left[E_{h,k}\right] \epsilon$
- $\epsilon > 0 \rightarrow$ no underflow (as k grows large)
- This result hinges on infinite ESS

The Role of ESS: Suboptimal Policies

- Policies that do not use the ESS, or use it only partially
- No ESS with adaptation: For an optimized fraction $0 \le \alpha^{h_s,h_t} \le 1$

$$\begin{cases} D_k = D^{h_s,h_t} \\ E_{s,k} = \alpha^{h_s,h_t} E_{h,k} \\ E_{t,k} = (1 - \alpha^{h_s,h_t}) E_{h,k} \end{cases}$$

- No ESS with no adaptation: $\alpha^{h_s,h_t} = \alpha$
- Source-only ESS
- Channel-only ESS
- ... non-separable policies

Numerical Results

• Source model: i.i.d. source $U_{k,i} \sim \mathcal{N}(0, D_{max})$ with i = 1, ..., Mmeasurement $\sqrt{H_{s,k}}U_{k,i} + Z_{k,i}$ where $Z_{k,i} \sim \mathcal{N}(0, 1)$ i.i.d. Rate-distortion-energy function is given [He and Wu 06]

$$f^{h_s}(D_k, E_{s,k}) = rac{1}{b} \log_2\left(rac{D_{max} - D_{mmse}}{D_k - D_{mmse}}
ight) \xi(E_{s,k}),$$

(indirect) rate-distortion function with $D_{mmse} = (h_s + 1/D_{max})^{-1}$, $\xi(T_{s,k}) = \zeta \max \left[(bE_{s,k}/E_{s,max})^{-1/\eta}, 1 \right]$, with $\zeta > 1$, $\eta > 1$ and $E_{s,max}$ being design parameters

- Communication model: AWGN with channel SNR $H_{t,k}$, $g^{h_t}(E_t) = \log(1 + h_t E_t)$
- $H_{s,k}$ and $H_{t,k}$ can take two possible values in $\mathcal{H}_s = \mathcal{H}_t = \{1, 10\}$ independently and with equal probability

Numerical Results

• Beta distribution for $E_{h,k}$ with mean $\mathbb{E}[E_{h,k}] = 0.5$

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$$E_{s,max}=1$$
, $b=1$ and $\eta=3/2$



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$$\eta = 3 (f^{h_s}(D_k, E_{s,k})$$
 less convex in $E_{s,k})$



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- Stability criterion does not provide any guarantee on the queue length (and thus, indirectly, on the delay)
- At the stability limit, the queue length becomes arbitrarily large
- Here we assume finite ESS and data queue
- Minimize expected total discounted cost ($0 \le \lambda < 1$ is the discount factor and $0 \le \gamma \le 1$)

$$\lim_{n\to\infty}\sum_{k=0}^n\lambda^k\left[\gamma\mathbb{E}[D_k]+(1-\gamma)\mathbb{E}[X_k]\right],$$

Dynamic programming

Numerical Results

• Data buffer length equal to 6, unit battery capacity, $D_k \in \{0.55, 0.75, 1\}$

•
$$p_{E_h}(0.5) = p_{H_t}(1) = p_{H_s}(1) = p_w$$



 Separable policy with two ESSs (assumes that the other encoder provides a constant and optimized rate in each slot)

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Multi-hop Network: Introduction



- Spatially correlated sources
 - -> Distributed source coding
 - -> In-network processing
- Interference
- Spatially correlated harvesting



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Source Coding Model



- Source state $\mathbf{O}(t)\in\mathcal{O}$ (finite) i.i.d.: joint spatial distribution of the sources $\sim\rho_{o_i}$
- ex.: $\mathcal{N}(\mathbf{0}, \mathbf{O}(t))$
- Source coding rate (bits per source symbol): $R_n(t) \leq R_{\max}$
- Distortion: $D_{\min} \leq D_n(t) \leq D_{\max}$, with $0 < D_{\min} \leq D_{\max} < \infty$

Source Coding Model: Distributed Source Coding



Trade rates of different users for fixed distortion levels

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Energy-Harvesting Source-Channel Coding

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Source Coding Model: Distributed Source Coding



• High-resolution rate-distortion constraints [Zamir Berger 99]:

$$\sum_{n \in \mathcal{X}} R_n(t) \ge h(\mathcal{X}, \mathbf{O}(t)) - \log_2 \left((2\pi e)^{|\mathcal{X}|} \prod_{n \in \mathcal{X}} D_n \right)$$

Source Coding Model: Distributed Source Coding



• Assume efficient distributed source codes: $P_c(R_n(t)) = \alpha_n R_n(t)$, $\alpha_n \ge 0$

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Transmission Model [Georgiadis et al 06]



- Transmission power (per channel use) on (n, m) link: $P_{n,m}(t)$
- Channel state $\mathbf{S}(t) \in \mathcal{S}$ (finite set) i.i.d. $\sim
 ho_{s_i}$
- Transmission rate $C_{n,m}(\mathbf{P}(t), \mathbf{S}(t))$ (bit per channel use) non decreasing in $P_{n,m}(t)$
- Interference is detrimental: $C_{n,m}(\mathbf{P}(t), \mathbf{S}(t)) \leq C_{n,m}(\mathbf{P}'(t), \mathbf{S}(t))$ if $P_{a,b}(t) = 0$ with $(a, b) \neq (n, m)$
- Ex: $\mu_{n,m}(t) = C_{n,m}(\mathbf{P}(t), \mathbf{S}(t)) = \log \left(1 + \frac{P_{n,m}(t)S_{n,m}(t)}{N_0 + \sum_{l \neq n} P_l(t)S_{l,n}(t)}\right)$



- Data received by other nodes: $\mu_{\mathit{in}}(t) = \sum_{m
 eq n} \mu_{m,n}(t)$
- Data transmitted: $\mu_{out}(t) = \sum_{n \neq m} \mu_{n,m}(t)$

$$U_n(t+1) \le \max \left\{ U_n(t) - \mu_{n,*}(t), 0 \right\} + \mu_{*,n}(t) + \frac{R_n(t)}{b}$$

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- Energy harvestable $H_n(t) \in \mathcal{H}$ (finite) i.i.d. $\sim \rho_{h_i}$
- Energy harvested: $0 \leq \tilde{H}_n(t) \leq H_n(t)$
- Battery evolution: $E_n(t+1) = E_n(t) P_n(t) P_c(R_n(t)) + \tilde{H}_n(t)$
- Energy availability constraints $P_n(t) + P_c(R_n(t)) \le E_n(t)$

Problem Definition

- Policy $\pi:=\{\pi(t)\}_{t\geq 1}$, where $\pi(t):=\{\mathsf{D}(t),\mathsf{R}(t),\mathsf{P}(t)\}_{\Theta(t)}$
- History of current and past states: $\Theta(t) = (\mathbf{U}(t'), \mathbf{E}(t'), \mathbf{S}(t'), \mathbf{O}(t'))_{t' \leq t}$
- Minimize over π

$$\limsup_{T\to\infty}\frac{1}{T}\sum_{t=1}^{T}\sum_{n}\mathbb{E}\left[f_{n}(D_{n}(t))\right]$$

- $f_n(D_n(t))$ convex and non-decreasing
- Strong stability constraint:

$$\limsup_{T\to\infty}\frac{1}{T}\sum_{t=1}^{T}\sum_{n}\mathbb{E}\left[U_{n}(t)\right]<\infty$$

• Energy availability and rate-distortion constraints

• By relaxing stability constraint to mean rate stability

$$\limsup_{T \to \infty} \frac{1}{T} \sum_{t=1}^{T} \sum_{n} \mathbb{E} \left[\mu_{*,n}(t) + \frac{R_n(t)}{b} \right] \leq \liminf_{T \to \infty} \frac{1}{T} \sum_{t=1}^{T} \sum_{n} \mathbb{E} \left[\mu_{n,*}(t) \right]$$

and the energy availability constraint as

$$\limsup_{T \to \infty} \frac{1}{T} \sum_{t=1}^{T} \sum_{n} \mathbb{E} \left[P_n(t) + P_c(R_n(t)) \right] = \liminf_{T \to \infty} \frac{1}{T} \sum_{t=1}^{T} \sum_{n} \mathbb{E} \left[\tilde{H}_n(t) \right]$$

constrained Markov decision process

- Optimal strategy is randomized stationary [Ross 95]
- Policy depends on $(\mathbf{S}(t), \mathbf{O}(t))$

• Using Lagrangian relaxation: For any V>0, $\lambda\in\mathbb{R}^{L(2^N-1)}_+$, $\nu\in\mathbb{R}^N_+$, $\chi\in\mathbb{R}^N$

$$VF_0^* \geq d(\lambda,
u, \chi)$$

with

$$d(\boldsymbol{\lambda},\boldsymbol{\nu},\boldsymbol{\chi}) = \sum_{o_i \in \mathcal{O}} \rho_{o_i} \sum_{s_j \in \mathcal{S}} \rho_{s_j} \sum_{h_k \in \mathcal{H}} \rho_{h_k} d_{o_i,s_j,h_k}(\boldsymbol{\lambda}^{(o_i)},\boldsymbol{\nu},\boldsymbol{\chi}),$$

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Lower Bound

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 with the definition $\mathit{d}_{o_i,s_j,h_k}(m{\lambda}^{(o_i)},
u,m{\chi})=0$

$$\inf_{\mathbf{R}^{(o_i)}, \mathbf{D}^{(o_i)}, \mathbf{P}^{(s_j)}, \tilde{\mathbf{H}}^{(h_k)}} \sum_{n} Vf_n(D_n^{(o_i)})$$

$$+ \sum_{m=1}^{2^N - 1} \lambda_m^{(o_i)} \left[h(\mathcal{X}_m, o_i) - \log\left((2\pi e)^{|\mathcal{X}_m|} \prod_n D_n^{(o_i)} \right) - \sum_{n \in \mathcal{X}} R_n^{(o_i)} \right]$$

$$+ \sum_{n} v_n \left[\frac{R_n^{(o_i)}}{b} + \mu_{*,n}(\mathbf{P}^{(s_i)}, s_i) - \mu_{n,*}(\mathbf{P}^{(s_i)}, s_i) \right]$$

$$+ \sum_{n} \chi_n \left[P_n^{(s_j)} + \alpha_n R_n^{(o_i)} - \tilde{H}_n^{(h_k)} \right],$$

under constraints $R_n^{(o_i)} \leq R_{\max}$, $D_{\min} \leq D_n^{(o_i)} \leq D_{\max}$ and $\tilde{H}_n^{(h_k)} \leq h_{n,k}$

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• Consider the problem:

minimize
$$\limsup_{T \to \infty} \frac{1}{T} \sum_{t=1}^{T} \mathbb{E}[f(x(t), \phi(t))]$$

s.t.
$$\begin{cases} \limsup_{T \to \infty} \frac{1}{T} \sum_{t=1}^{T} \mathbb{E}[Q(t)] \le \infty \\ x(t) \in \mathcal{X}_{\phi(t)} \end{cases}$$

• $\phi(t)$ (state) i.i.d. • Queue: $Q(t+1) = \max\{Q(t) + g(x(t), \phi(t)), 0\}$

Lyapunov Optimization [Georgiadis et al 06] [Neely 10]

- Lyapunov function: $L(t) = \frac{1}{2}Q(t)^2$
- Average drift: $\Delta(t) = \mathbb{E}[L(t+1) L(t)|Q(t)]$
- Minimize

$$\Delta(t) + V\mathbb{E}[f(x(t),\phi(t))|Q(t)]$$

$$\leq B + Q(t)\mathbb{E}[g(x(t),\phi(t))|Q(t)] + V\mathbb{E}[f(x(t),\phi(t))|Q(t)]$$

Minimize

$$Q(t)g(x,\phi) + Vf(x,\phi)$$
 s.t. $x \in \mathcal{X}_{\phi}$

(no need to know statistics of state $\phi(t)$)

Lyapunov Optimization [Georgiadis et al 06] [Neely 10]

• Relax the stability constraint: $\phi^* \leq V\!F^*$, where ϕ^* optimal solution of

$$\begin{array}{l} \underset{T \to \infty}{\text{minimize}} \limsup_{T \to \infty} \frac{1}{T} \sum_{t=1}^{T} V \mathbb{E}[f(x(t), \phi(t))] \\ \text{s.t.} & \left\{ \begin{array}{l} \limsup_{T \to \infty} \frac{1}{T} \sum_{t=1}^{T} \mathbb{E}[g(x(t), \phi(t))] \leq 0 \\ x(t) \in \mathcal{X}_{\phi(t)} \end{array} \right. \end{array}$$

Lagrange dual (weak duality): d^{*} ≤ φ^{*} ≤ VF^{*}, where d^{*} is the solution of

 $\begin{array}{l} \mbox{minimize } V\mathbb{E}[f(x(t),\phi(t))|Q(t)] + \lambda\mathbb{E}[g(x(t),\phi(t))|Q(t)] \\ \mbox{s.t. } x(t) \in \mathcal{X}_{\phi(t)} \end{array}$

Lyapunov Optimization [Georgiadis et al 06] [Neely 10]

• For the considered policy

$$\Delta(t) + V\mathbb{E}[f(x(t), \phi(t))|Q(t)]$$

$$\leq B + Q(t)\mathbb{E}[g(x(t), \phi(t))|Q(t)] + V\mathbb{E}[f(x(t), \phi(t))|Q(t)]$$

$$\leq B + VF^*$$

• Averaging over Q(t) and summing over t

$$\frac{1}{T}L(T) + \frac{1}{T}\sum_{t=1}^{T} V\mathbb{E}[f(x(t), \phi(t))]$$

$$\leq B + VF^*$$

• By taking the limit, $\limsup_{T\to\infty} \frac{1}{T}\sum_{t=1}^T \mathbb{E}[f(x(t),\phi(t))] \leq \frac{`B}{V} + F^*$

- Use Lyapunov based optimization with queues $\{U_n(t)\}$ and $\{E_n(t)\}$
- Perturbed Lyapunov function ¹/₂(E_n(t) θ_n)²
 ... drive the battery to a desired value so as to avoid underflow [Huang Neely 10]

Fix a weight $\theta = [\theta_1, \dots, \theta_N] \in \mathbb{R}^N_+$ and a parameter V > 0. At each time-slot t,

- Energy Harvesting: For each node n, if $(E_n(t) \theta_n) < 0$ set $\tilde{H}_n(t) = H_n(t)$; otherwise, $\tilde{H}_n(t) = 0$;
- *Rate-Distortion Optimization:* Choose the rate vector **R**(*t*) and the distortion levels **D**(*t*) as

minimize
$$\sum_{n} \left[U_n(t) r_n - (E_n(t) - \theta_n) \alpha_n r_n + V f_n(d_n) \right]$$

subject to the rate-distortion region constraint and to $0 \le r_n \le R_{\max}$ and $D_{\min} \le d_n \le D_{\max}$; • Power Allocation: Define the weight of a link (n, m) as

$$W_{n,m}(t) = (U_n(t) - U_m(t) - \delta)^+,$$

where $\delta = \mathit{I}_{\max} \mu_{\max} + \mathit{R}_{\max}$, and choose the power matrix $\mathbf{P}(t)$ as

$$\underset{\mathbf{p}}{\text{maximize}} \sum_{n} (E_{n}(t) - \theta_{n}) p_{n} + \sum_{m \neq n,m} W_{n,m}(t) C_{n,m}(\mathbf{p}, \mathbf{S}(t)),$$

where $p_n = \sum_{m \neq n} p_{n,m}$, subject to constraints $0 \le p_{n,m} \le P_{\max}$; • Queues Update

- Rate-Distortion Optimization and Power Allocation require centralized optimization
- Distributed implementation of Power Allocation [Hoepman 04][Chen et al 06]
- Distributed implementation of Rate-Distortion Optimization: dual decomposition and subgradient

Performance Analysis

- Define the parameters $\beta_n = \min\{\alpha_n, 1\}$, $\gamma_n = \sup_{D_{\min} \le d_n \le D_{\max}} \left[\frac{f_n(d_n) - f_n(D_{\max})}{\log(d_n/D_{\max})} \right] < \infty$ • With $\theta_n = \alpha_n(H_{\max} + R_{\max}) + \frac{\gamma_n}{\beta_n}V + I_{\max}P_{\max}$,
- 1 Bounded energy and data queues for all t:

$$0 \leq E_n(t) \leq heta_n + H_{\max},$$

and $0 \leq U_n(t) \leq lpha_n H_{\max} + \gamma_n V + R_{\max}$

2 Cost function:

$$\sum_{n=1}^N f_n^\pi \leq F_0^* + \frac{B}{V},$$

where $B = N \left(\mu_{\max}(\mu_{\max} + R_{\max}) + R_{\max}^2/2 \right) + N/2(H_{\max}^2 + \alpha_n^2 R_{\max}^2 + P_{\max}^2 + 2\alpha_n R_{\max} P_{\max}) + N\delta I_{\max} \mu_{\max}$

3 No battery underflow: If $P_n(t) > 0$ and/or $R_n(t) > 0$, we have that

$$E_n(t) \ge \alpha_n R_{\max} + I_{\max} P_{\max}$$

Numerical Results



- $f_n(D_n) = D_n$ (sum-distortion)
- $S_{n,m}(t) \sim U(0,1), H_n(t) \sim U(0,1)$
- Gaussian sources with correlation ho

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$$\alpha_n = 2$$
, $P_{\max} = 3$

Numerical Results: Impact of V



• *ρ* = 0.5

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Numerical Results: Impact of V



• $\rho = 0.5$

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Numerical Results: Impact of Source Correlation



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Numerical Results: Impact of Source Correlation



- Wireless sensor networks are a key component of cyber-physical systems
- Energy-harvesting enables new applications of wireless sensor networks
- Energy requirements of source and channel coding
- Point-to-point With infinite data and energy buffers: separable policy is optimal
- With sufficiently large data and energy buffers: separable policy is approximately optimal
- Trade-off buffer sizes and performance