both via a multibody motorcycle simulator and with an instrumented vehicle equipped with a yaw gyroscope and a steering potentiometer. The proposed observers allow reducing the number of sensors needed to implement an innovative algorithm to control a semi-active steering damper. Promising results have been obtained in simulation and in the preliminary experimental tests.

Current research is being focused on the experimental validation of the single-sensor LPV and LTI mixed dynamic control strategies. To do that, significant experimental tests must be devised to excite both the weave and wobble modes without causing danger to the rider.

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Spectrum Leasing via Cooperation With Multiple Primary Users

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***Abstract*—Within the paradigm of spectrum leasing via cooperation, primary (licensed) nodes can lease some of the owned spectral resources to secondary (unlicensed) users in exchange for cooperation. Secondary users, in turn, set a minimal quality-of-service (QoS) requirement on the spectrum leased as a precondition for cooperation. Previous work assumed that a single primary user makes spectrum-leasing decisions in the presence of possibly multiple secondary users. In this paper, the analysis is extended to accommodate multiple primary users by adopting the framework of generalized Nash equilibrium (GNE) problems. Accordingly, multiple primary users, each owning its own spectral resource, compete for the cooperation of the available secondary users under a shared constraint on all spectrum-leasing decisions set by the secondary QoS requirements. A general formulation of the problem is given, and solutions are proposed with different signaling requirements among the primary users. Then, application of the framework is discussed for a practical example that includes communication over fading channels with retransmissions. Numerical results bring insight into the advantages of spectrum leasing and of the effectiveness of the proposed solutions.**

***Index Terms*—Cognitive radio, cooperative systems, generalized Nash equilibrium (GNE), spectrum leasing, variational inequality (VI).**

I. Introduction

Consider the scenario where multiple primary (licensed) users PT_m, $m \in \{1, \ldots, M\}$, communicating over *orthogonal* spectral resources, coexist with multiple secondary (unlicensed) users ST_n, $n \in \{1, \ldots, N\}$, as shown in Fig. 1(a) (for $M = 2$ and $N = 1$). Within the paradigm of spectrum leasing via cooperation [1], [2] (see also related ideas in [3]–[6]), the primary nodes can lease some of the owned spectral resources to secondary nodes in exchange for cooperation. A secondary node ST_n accepts to cooperate in forwarding primary traffic only if offered enough spectral resources to satisfy its own quality-of-service (QoS) requirement. For instance, in Fig. 1, user ST_1 accepts to forward data for PT_1 and PT_2 [see Fig. 1(c)] upon being offered fractions, e.g., α_1 and α_2, of the time slots (or bandwidths) owned by PT_1 and PT_2, respectively, for its own transmission [see Fig. 1(d)].

Previous work [1], [2] assumed that a *single* primary user is present that makes spectrum-leasing decisions in the presence of possibly multiple secondary users (i.e., $M = 1, N > 1$). In this paper, the analysis is extended to accommodate multiple primary users, i.e.,

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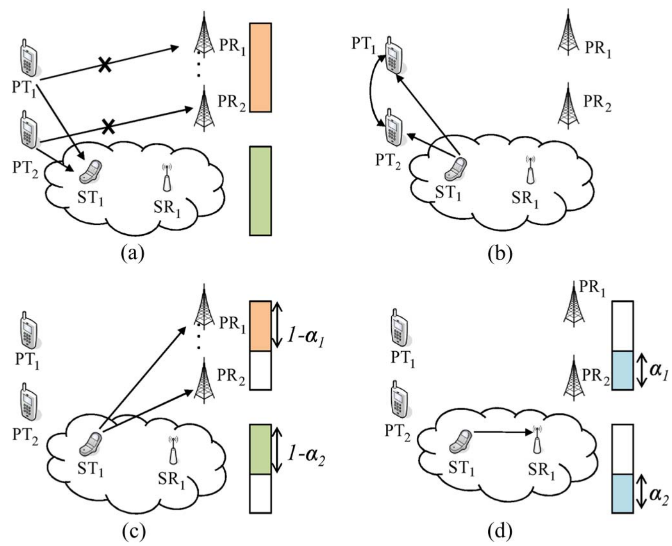

Fig. 1. Spectrum leasing with multiple ($M = 2$) primary users and $N = 1$ secondary user, where the primary users operate over orthogonal spectral resources.

$M > 1$, by adopting the framework of generalized Nash equilibrium (GNE) problems. With multiple primary users, the QoS requirements of the secondary nodes impose a shared constraint on the spectrum-leasing decisions of the primary nodes. For instance, in the example in Fig. 1, ST_1 may request a QoS corresponding to a fraction $q \geq 0$ of a time slot (or bandwidth). Therefore, as long as enough spectrum is collectively leased by PT_1 and PT_2, i.e., $\alpha_1 + \alpha_2 \geq q$, ST_1 will be willing to cooperate with the primary users.

The presence of shared secondary QoS constraints ties the decisions of different primary users, despite the fact that their operation is in orthogonal spectral resources. We model this scenario as a GNE problem, which is a generalization of standard Nash Equilibrium (NE) problems (e.g., see [7]), that includes joint constraints on the actions of the players (see Section II). GNE solutions are discussed in Section III, which different tradeoffs between performance and signaling requirements among the primary users. Moreover, an application of the framework is proposed for a scenario, such as in Fig. 1, that includes communication over fading channels with retransmissions, which are discussed in Section IV. Numerical results are also provided in Section IV to obtain insight into the performance of the proposed solutions.

II. System Model

We consider M primary users PT_m, $m \in \{1, \ldots, M\}$, each active on a separate spectral resource, and $N = 1$ secondary user ST, with a minimum QoS requirement q. Note that we have removed the subscript for the secondary user for simplicity of notation. Moreover, the analysis here can be extended to the general case of $N > 1$ secondary users under assumptions that will be discussed later. Each primary user PT_m optimizes a spectrum-leasing parameter α_m, belonging to a nonempty, compact, and convex set $\mathcal{A}_m$, thus deciding the amount of spectrum to be leased to ST. We will assume $\mathcal{A}_m = [0, 1]$ for simplicity, so that α_m accounts for the fraction of the spectral resources that PT_m leases to ST. The idea is that the remaining fraction $(1 - \alpha_m)$ of the spectral resources will be used by ST to cooperate with the primary user PT_m, as shown in Fig. 1(c) and (d). Therefore, in general, each PT_m is interested in minimizing a cost function $f_m(\alpha_m)$, which is strictly monotonically increasing in α_m, i.e., the amount of leased spectrum, and is independent of all other α_i with $i \in \{1, \ldots, M\}, i \neq m$. Independence of the other α_i follows from the orthogonality of the spectral resources of all primary users. We assume that $f_m(\alpha_m)$ is continuously differentiable and quasi-convex on set $\mathcal{A}_m$. Specific examples will be given in Sections III-C and IV.

The secondary user ST accepts to cooperate with the primary users as long as it receives enough leased spectrum. The QoS requirement is parameterized by a value $q > 0$ and imposes a joint constraint on all spectrum decisions $\boldsymbol{\alpha} = (\alpha_1, \ldots, \alpha_M)$ as

$$g(\boldsymbol{\alpha}, q) \leq 0 \tag{1}$$

where $g(\boldsymbol{\alpha}, q)$ is a convex and continuously differentiable function in $\boldsymbol{\alpha} \in \mathcal{A}_1 \times \cdots \times \mathcal{A}_M$ for each $q > 0$. For instance, following the example given in Section I, a possible choice for $g(\boldsymbol{\alpha}, q)$ is $g(\boldsymbol{\alpha}, q) = -\sum_{m=1}^{M} \alpha_m + q$, which guarantees from (1) that the sum of all fractions of leased spectra, i.e., $\sum_{m=1}^{M} \alpha_m$, is larger than the QoS target q. Overall, each PT_m attempts to solve the following problem:

$$\begin{aligned}
&\underset{\alpha_m \in \mathcal{A}_m}{\text{minimize}} \ f_m(\alpha_m) \\
&\text{subject to } g(\boldsymbol{\alpha}, q) \leq 0 \\
&\alpha_m = \arg \max_{\alpha_m \in \mathcal{A}_m} f_m(\alpha_m, \boldsymbol{\alpha}_{-m}) \\
&\text{subject to } g(\boldsymbol{\alpha}, q) \leq 0.
\end{aligned} \tag{2}$$

Problems (2) for $m \in \{1, \ldots, M\}$ are coupled by the shared secondary QoS constraint $g(\boldsymbol{\alpha}, q)$, and their collection constitutes a GNE problem [8]. A GNE problem generalizes the classical notion of NE problems due to the presence of joint constraint (1).

A practical application of this framework is discussed in Section IV. It is noted that the generalization of (2) to the case of multiple secondary users is straightforward as long as each secondary user ST_n has a fixed QoS requirement, e.g., q_m. In fact, the extension requires including a spectrum-leasing decision, e.g., α_{mn}, for each pair PT_m–ST_n and one QoS constraint for each ST_n in (2). In the more general case where QoS requirements q_m can be optimized by the secondary users in a strategic fashion, the problem is more complex. Related scenarios, with $M = 1$, were addressed in [1] and [2]. In the following, we discuss how the primary users can perform the desired minimization in (2).

III. Spectrum-Leasing Strategies

As discussed, each primary user is interested in solving problem (2) to maximize the advantages accrued from spectrum leasing. In performing this optimization, there is clearly a conflict among the primary users due to shared QoS constraint (1). In the following, we discuss two classes of solutions, namely, GNE and variational inequality (VI) solutions. As it will be shown, these two classes strike different tradeoffs between signaling requirements among the primary users and overall system performance.

A. GNE Strategies

Define by $\boldsymbol{\alpha}_{-m}$ the vector obtained from $\boldsymbol{\alpha}$ by removing α_m. For a fixed $\boldsymbol{\alpha}_{-m}$, let $\mathcal{S}_m(\boldsymbol{\alpha}_{-m}) \subseteq \mathcal{A}_m$ be the set of solutions, possibly empty, of problem (2). A GNE $\boldsymbol{\alpha}$ is any vector such that $\alpha_m \in \mathcal{S}_m(\boldsymbol{\alpha}_{-m})$ for all $m \in \{1, \ldots, M\}$. In other words, a GNE $\boldsymbol{\alpha}$ is such that any mth entry α_m solves problem (2) given the other entries $\boldsymbol{\alpha}_{-m}$. In other words, a GNE $\boldsymbol{\alpha}$, generalizing the concept of NE, corresponds to a solution that discourages unilateral deviations by the players (here, primary users) under joint constraint (1). Given the assumptions made, it can be proved that the GNE problem at hand admits at least one GNE [6, Th. 4.1]. As it will be discussed in the following, in fact, there are typically many GNEs.

The question arises as to how the primary users can select spectrum-leasing decisions α_m that are GNEs. The following distributed algorithm, which is referred to as Algorithm GNE, can be proved to have the property that, if a limit point $\boldsymbol{\alpha}$ is reached, that point $\boldsymbol{\alpha}$ is a GNE [8]: (s.0) Choose a feasible initial vector $\boldsymbol{\alpha}^j = [\alpha_1^j \cdots \alpha_M^j] \in \mathcal{A}_1 \times \cdots \times \mathcal{A}_M$, with iteration index $j = 0$. (s.1) If $\boldsymbol{\alpha}^j$ satisfies a suitable termination criterion, stop and take $\boldsymbol{\alpha} = \alpha^j$. (s.2) For $m \in \{1, \ldots, M\}$, find a solution $\alpha_m^{j+1} \in \mathcal{S}_m(\boldsymbol{\alpha}_{-m}^j)$. (s.3) Set $\boldsymbol{\alpha}^{j+1} = [\alpha_1^{j+1} \cdots \alpha_M^{j+1}]$ and $j \leftarrow j+1$, then go to (s.1).

The preceding algorithm, which is an instance of Gauss–Seidel iterations, requires the secondary users to communicate the QoS value q to all primary users at the beginning of the decision process. Then, each iteration requires the primary users to exchange their previous decisions $\boldsymbol{\alpha}^j$.

B. VI Strategies

GNE solutions are simple to obtain using the previously discussed "Algorithm GNE" (although convergence, in principle, is not guaranteed). However, there are typically many GNEs, and most GNEs typically correspond to solutions that are rather inefficient from the standpoint of the system performance [9]. A subclass of GNEs that turns out to have desirable performance is given by the so-called VI solutions. A point $\boldsymbol{\alpha}$ is a VI solution if there exists a parameter $\mu \geq 0$ such that $\boldsymbol{\alpha}$ is an NE of the strategic game set by the simultaneous solution of the problems as follows:

$$\underset{\alpha_m \in \mathcal{A}_m}{\text{minimize}}\, f_m(\alpha_m) + \mu g(\boldsymbol{\alpha}, q) \tag{3}$$

for $m \in \{1, \ldots, M\}$, and conditions $\mu g(\boldsymbol{\alpha}, q) = 0$ and $g(\boldsymbol{\alpha}, q) \leq 0$ are satisfied. Parameter μ can be interpreted as a "price" variable that inflicts an additional cost to the primary users in case constraint $g(\boldsymbol{\alpha}, q) \leq 0$ is not satisfied. It can be shown that, in general, VI solutions are also GNEs, but the converse is not true. Moreover, given our assumptions, at least one VI solution always exists [9]. In important cases, such as when cost functions $f_m(\alpha_m)$ are strongly convex,[1] the VI solution can be also proved to be unique [9].

We will see in the following that a VI solution can be attained by a distributed algorithm with stronger signaling requirements than "Algorithm GNE." We now show, however, that VI solutions have very desirable performance in the model at hand. In fact, since the cost function $f_m(\alpha_m)$ of each primary user is independent of all other primary users' decision variables, it can be shown that a VI solution is also a solution of the following centralized problem:

$$\underset{\boldsymbol{\alpha} \in \mathcal{A}_1 \times \cdots \times \mathcal{A}_M}{\text{minimize}} \sum_{m=1}^{M} f_m(\alpha_m)$$

$$\text{subject to } g(\boldsymbol{\alpha}, q) \leq 0. \tag{4}$$

This can be proved by noting that the Karush–Kuhn–Tucker (KKT) optimality conditions [10] of problem (4) coincide with the collection of the KKT optimality conditions of problems (3) along with the aforementioned additional conditions, i.e., $\mu \geq 0$, $\mu g(\boldsymbol{\alpha}, q) = 0$, and $g(\boldsymbol{\alpha}, q) \leq 0$ (see also [11]).[2] Problem (4) corresponds to the centralized optimization of spectrum leasing that minimizes the sum of all costs. The fact that a VI solution also solves (4) implies that VI solutions are efficient in the sense of minimizing the sum cost.

The following algorithm, which is referred to as Algorithm VI, is known to converge to a VI solution: (s.0) Choose an initial price $\mu^j \geq 0$, with iteration index $j = 0$, and a step size $\tau > 0$. (s.1) If $\boldsymbol{\alpha}^j$ satisfies a suitable termination criterion, stop and take $\boldsymbol{\alpha} = \boldsymbol{\alpha}^j$. (s.2) Find an NE $\boldsymbol{\alpha}^j = [\alpha_1^j \cdots \alpha_M^j]$ of the game defined by (3) for $m \in \{1, \ldots, M\}$, where $\mu = \mu^j$. This can be done using Gauss–Seidel iterations as per "Algorithm GNE" with (3) in lieu of (2). (s.3) Update the price according to the subgradient rule $\mu^{j+1} = [\mu^j - \tau(g(\boldsymbol{\alpha}^j, q))]^+$. (s.4) Set $j \leftarrow j+1$, then go to (s.1).

"Algorithm VI" requires two nested loops, instead of a single loop as for "Algorithm GNE." The outer loop updates price μ, whereas the inner loop calculates the required NE for a fixed price. This double loop requires more signaling among the primary users to converge, in a proportion that depends on the number of iterations of the outer loop necessary for convergence. In fact, the inner loop of "Algorithm VI" requires approximately the same number of iterations of "Algorithm GNE." Note that price μ^j can be either set by the secondary user at the beginning of each outer iteration or, more practically, calculated by each primary user based on the knowledge of $\boldsymbol{\alpha}^j$ and q.

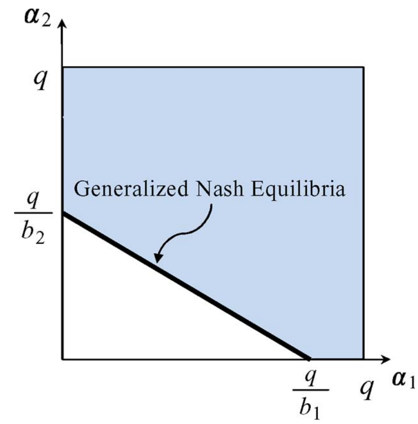

Fig. 2. Feasible set and GNE for the case of linear QoS secondary constraints and $M = 2$ primary users.

C. Linear QoS Secondary Constraints and $M = 2$

Here, we specialize the aforementioned results to the special case where we have $M = 2$ primary users and the QoS function is linear as in $g(\alpha_1, \alpha_2, q) = -b_1\alpha_1 - b_2\alpha_2 + q$, where $\alpha_1, \alpha_2 \in \mathcal{A}_1 = \mathcal{A}_2 = [0, 1]$, $b_1, b_2 \geq 1$, and $0 \leq q \leq 1$. QoS constraint (1) is thus

$$b_1\alpha_1 + b_2\alpha_2 \geq q \tag{5}$$

so that parameters b_1 and b_2 weight the usefulness of spectrum leased by PT_1 and PT_2, respectively, for ST. Moreover, spectrum-leasing parameters α_1 and α_2 represent the fraction of spectral resources leased to ST by PT_1 and PT_2, respectively. As an example, if the channel from ST toward its destination is in better conditions on the spectrum owned by PT_1, we have $b_1 \geq b_2$. A linear QoS constraint

[1]A continuously differentiable function f_m is said to be strongly convex on $\mathcal{A}_m$ if $(\nabla f_m(u) - \nabla f_m(v))^T(u - v) \geq m\|u - v\|^2, m > 0,\ \forall u, v \in \mathcal{A}_m$

[2]In other words, $\sum_{m=1}^{M} f_m(\alpha_m)$ is a potential function for the game at hand (e.g., see [7])

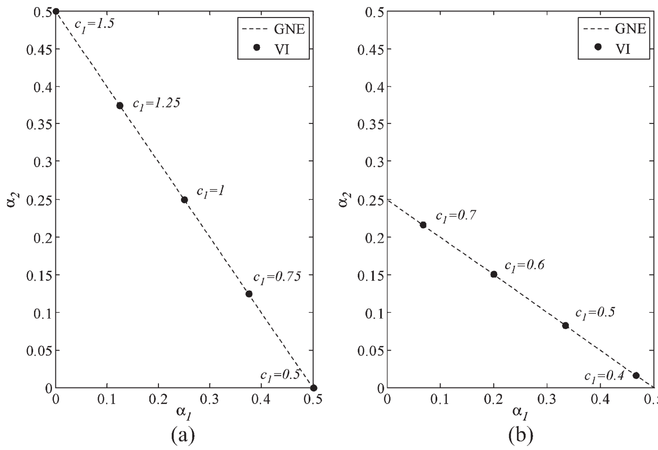

Fig. 3. VI solutions for the example of linear QoS secondary constraints and $M = 2$ primary users different values of c_1 and (a) $b_2 = 1$ and (b) $b_2 = 2$ ($b_1 = 1, c_2 = 1, q = 0.5$).

is meaningful since many metrics such as achievable rates are indeed linear in the fraction of time ST is allowed to transmit.

We now characterize the GNE and VI solutions for this example for generic cost functions satisfying our general assumptions. The set of all feasible solutions $\boldsymbol{\alpha}$ satisfying the QoS constraint and $\alpha_1, \alpha_2 \in [0, 1]$ is the shaded region in Fig. 2. If $q = 0$, due to the assumption of strict monotonicity of the cost functions, the only solution of GNE and VI is easily shown to be $\alpha_1 = \alpha_2 = 0$. With $q > 0$, there are an infinite number of GNEs, which are given by all the points on the segment shown in Fig. 2 or, equivalently, all points of the form $(\alpha_1, \alpha_2) = (\alpha_1, (q - b_1\alpha_1/b_2)$ for $\alpha_1 \in [0, q/b_1]$. This is because, for any point $\boldsymbol{\alpha}$ on this segment, no primary user can further reduce its cost while still satisfying the secondary QoS constraint (recall that the cost function $f_m(\alpha_m)$ is strictly increasing in α_m). VI solutions are given instead by the point or points on this segment that minimize (4), i.e., the sum cost function $f_1(\alpha_1) + f_2(\alpha_2)$.

For a more specific example, assume that the rate that the secondary is able to provide for PT_m via cooperation is proportional to the fraction of time that is not leased, namely, to $(1 - \alpha_m)$. A fairly standard cost function is $f_m(\alpha_i) = -c_m \log(1 - \alpha_m)$, where $c_m \geq 0$ are constants, which for $c_m = 1$ leads to the so-called proportional fairness solution when solving (4). Since this cost is strongly convex, as aforementioned, there is only one VI solution. Using the KKT conditions of problem (4), the VI solution is easily found as

$$\alpha_1 = \max\left\{0, \min\left\{\left(1 - \frac{b_2 c_1}{b_1 c_2} + q\right) \Big/ \left(1 + \frac{b_1}{b_2}\right), \frac{q}{b_1}\right\}\right\} \tag{6}$$

and $\alpha_2 = (q - b_1\alpha_1)/b_2$, and is shown in Fig. 3 for different values of c_1 and fixed $b_1 = 1$, $c_2 = 1$, $q = 0.5$, and $b_2 = 1$ in Fig. 3(a), whereas $b_2 = 2$ in Fig. 3(b). It can be shown that, as c_1 increases, the VI solution moves from $(q/b_1, 0)$, where only PT_1 leases the spectrum, to $(0, q/b_2)$, where only PT_2 leases the spectrum.

IV. Spectrum Leasing via HARQ

Here, we provide an application of the framework discussed so far.

A. Setting

Consider the system in Fig. 1 with $N = 1$ secondary user ST and $M = 2$ primary users PT_1 and PT_2. Primary users PT_1 and PT_2 are active in their dedicated channels (e.g., different frequencies), and their transmission to their respective destinations PR_1 and PR_2 is slotted and not necessarily synchronous. Nevertheless, we number the time slots for the primary users so that time slots with the same index take place sufficiently close in time to enable the protocol discussed in the following. In the first slot, primary transmitters PT_1 and PT_2 communicate at fixed rates R_1 and R_2 (bit/channel use) and with powers P_1 and P_2, respectively, directly to their intended primary receivers, as shown in Fig. 1(a). If either direct link is in outage, a retransmission takes place, and the corresponding primary transmitter may decide to grant the retransmission slot to ST in exchange for cooperation. Specifically, with spectrum leasing, any primary link PT_m that was in outage in the previous slot offers a fraction of duration $0 \leq \alpha_m \leq 1$ to the secondary link for secondary transmission as an incentive for cooperation [see Fig. 1(d)]. The remaining fraction $1 - \alpha_m$ is utilized by the secondary user for relaying primary traffic [see Fig. 1(c)]. The aggregate of all primary offers must satisfy a linear secondary QoS requirement q as in (5) for the given b_1 and $b_2 = 1$ in order for the secondary link to accept cooperation. Cooperation takes place via decode-and-forward, as explained in the following.

Fading channels are assumed, and the power gain for the link between a transmitter i and a receiver j is $g_{ij} = |h_{ij}|^2 d_{ij}^{-\eta}$, where h_{ij} is the complex Rayleigh fading channel gain between nodes i and j; d_{ij} is the distance between nodes i and j; and η is the path-loss exponent. Fading channels are constant during the transmission slot but independently change from one slot to the other. If any packet is not successfully decoded, a negative-acknowledge message requesting retransmission is broadcast. As introduced earlier, primary links employ type-I hybrid automatic repeat request (HARQ), whereby copies of the same packet are retransmitted and decoded without leveraging previous transmissions, with a maximum number of retransmissions of one. We define $P_{\text{out},ij} = 1 - \exp(-(2^{R_i} - 1/g_{ij}P_i))$ as the outage probability of link $i - j$ when transmission takes place with rate R_i (bit/channel use) and power P_i. We will use m as the label to identify the mth primary link, i.e., either PT_m or PR_m, and s to identify ST, so that $P_{\text{out},ms}$ is the outage probability on the link between PT_m and ST and that $P_{\text{out},mm}$ is the outage probability of the direct link PT_m–PR_m. Note that the outage probability $P_{\text{out},sm}$ between ST and PR_m depends on the fraction α_m leased to ST. This will be explicitly denoted by

$$P_{\text{out},sm}(\alpha_m) = 1 - \exp\left(-\frac{2^{\frac{R_m}{1-\alpha_m}} - 1}{g_{ij}P_i}\right).$$

We distinguish four cases. First, both primary packets are correctly received in the first slot. In this case, no retransmission is required, and a new transmission begins in the next slot. Second, only primary PT_2 is in outage in the first slot. If ST was able to decode PT_2's packet in the first slot, in the second slot of PT_2, ST is assigned a fraction $\alpha_2 = q/b_2$ of the spectrum for its own transmission [to satisfy (5)] and retransmits the primary packet in the remaining fraction $1 - \alpha_2$. If instead ST was not able to decode PT_2's packet, PT_2 performs retransmission. PT_1, which was not in outage in the first slot, is allowed to send a new packet in its second slot. Third, only PT_2 is in outage in the first slot. The protocol proceeds as for the second case but with the roles of PT_1 and PT_2 reversed. Fourth, both primary users are in outage in the first slot. If ST was able to decode both packets from the signal received in the first slot, ST can help both primary users in the second slot provided that its QoS constraint (5) is satisfied. The fractions (α_1, α_2) leased by PT_1 and PT_2 in the second slot are decided based on either a GNE or a VI solution, as further discussed in the following.

If instead ST was able to decode only one packet, e.g., that of PT_m, we operate as in the aforementioned second and third cases, except that the other primary will perform retransmission. If instead ST did not decode any packet, retransmissions are performed by the primary users.

B. GNE Problem Formulation

As explained earlier, it remains to be discussed how leased fractions (α_1, α_2) are calculated for the case where both primary users are in outage (as in the fourth case) and how the secondary is able to decode both packets in the first slot. Since QoS constraint (5) must be satisfied, PT_1 and PT_2 face a GNE problem (2), where we need to specify cost functions $f_m(\alpha_m)$. We will show that the choice

$$f_m(\alpha_m) = (1 - P_{\mathrm{out},ms})P_{\mathrm{out},sm}(\alpha_m) \tag{7}$$

is well justified and leads to desirable performance. Note that $f_m(\alpha_m)$ satisfies our general assumptions and is, in particular, quasi-convex [10, p. 95]. With this choice, parameters (α_1, α_2) at hand will be chosen as either a GNE or a VI solution of problem (2) with QoS constraint (5). Note that since (7) only depends on the channel statistics, primary and secondary nodes can agree on (α_1, α_2) in advance, by running either "Algorithm GNE" or "Algorithm VI," and keep the same decision for as long as the channel statistics remain the same. This protocol provides a generalization of the HARQ-based protocol studied in [2] to the setting with multiple primary users.

To further discuss (7), let us calculate the average primary system throughput $T_P = E[\text{Packets}]/E[S]$, where $E[\text{Packets}]$ is the average number of packets successfully received by the primary receivers, and $E[S]$ is the average number of slots. This can be easily calculated from the description of system (8), shown at the bottom of the page, where

$$P_1 = (1 - P_{\mathrm{out},11})(1 - P_{\mathrm{out},22}) \tag{9a}$$

$$P_2 = (1 - P_{\mathrm{out},11})P_{\mathrm{out},22} \tag{9b}$$

$$P_3 = P_{\mathrm{out},11}(1 - P_{\mathrm{out},22}) \tag{9c}$$

$$P_4 = P_{\mathrm{out},11}P_{\mathrm{out},22} \tag{9d}$$

are the probabilities of the events of the first, second, third, and fourth cases, respectively, as discussed earlier.

For instance, P_1 is the probability that packets of PT_1 and PT_2 are successfully received, and P_2 is the probability that PT_1's packet is successfully received, whereas PT_2's packet is not. To interpret (8), note that, for instance, the second term accounts for the average number of packets successfully delivered conditioned on the aforementioned second case taking place. In fact, the four terms in the sum multiplying P_2 are, respectively, the probability that PT_1's packet was successfully decoded in the first slot, which is equal to 1; the probability of successful transmission of PT_1's packet on the $\mathrm{PT}_1 - \mathrm{PR}_1$ link in the second slot, which is equal to $(1 - P_{\mathrm{out}11})$; the probability that PT_2's packet was decoded and is being relayed by ST, which happens with probability $(1 - P_{\mathrm{out},2s})(1 - P_{\mathrm{out},s2}(q/b_2))$; and the probability that PT_2's packet was not decoded by ST but was being retransmitted by PT_2, which is equal to $P_{\mathrm{out},2s}(1 - P_{\mathrm{out},22})$. Moreover, parameters (α_1, α_2) are obtained as GNE or VI solutions, as explained earlier.

From (8), it can be shown that solving centralized problem (4) with cost functions (7) leads to the maximization of throughput (8). This is because the only term that depends on $\boldsymbol{\alpha}$ in (8) is $(1 - P_{\mathrm{out},2s})(1 - P_{\mathrm{out},s2}(\alpha_2)) + (1 - P_{\mathrm{out},1s})(1 - P_{\mathrm{out},s1}(\alpha_1))$, whose minimization is equal to problem (4) with cost functions (7). This implies that VI solutions, given the discussion in Section III-B, maximize the throughput. The same cannot be generally said about GNE solutions, which is instead given by all points on the segment in Fig. 2, as explained in Section III-C.

Remark: We recall that the secondary QoS requirement q entails that any time the spectrum is lease, and thus secondary cooperation takes place, ST is guaranteed a QoS of q.

C. Numerical Results

To provide some numerical insight, assume that PT_1, PR_1, PT_2, and PR_2 have fixed locations in an $x-y$ plane at (0,0.25), (0,−0.25), (0.5,0.25), and (0.5, −0.25), respectively. Let d_s be the x-coordinate of ST and assume that it moves on the x-axis along with SR with a fixed distance between them. Assume fixed transmit powers $P_1 = P_2 = P_s = 1$, fixed rates $R_1 = R_2 = 2$, and path-loss exponent $\eta = 3$. We obtain the GNE and VI solutions from "Algorithm GNE" and "Algorithm VI," respectively, by choosing random initialization and averaging over the outcomes.

Fig. 4 plots the average primary system throughput T_P (8) versus ST's location d_S for GNE and VI for $b_1 = b_2 = 1$ and different values of the secondary QoS $q = 0$, 0.25, 0.5, and 1. We compare the performance with that obtained with no spectrum leasing (NSL), which corresponds to using only direct (re)transmissions (i.e., setting $P_{\mathrm{out},1s} = P_{\mathrm{out},2s} = 1$). First, it is observed that, for QoS q, sufficiently small (e.g., $q = 0.25$) spectrum leasing provides very relevant performance gains for the primary users but only as long as the location of ST is in the vicinity of the two primary users (e.g., $-0.25 \leq d_s \leq 0.75$) so that ST is able to cooperate with both users. This way, both users can share the burden of satisfying the secondary QoS constraints, and secondary cooperation is still advantageous, despite the fact that ST is not in the best position for neither PT_1 nor PT_2. If instead QoS constraint q is large (e.g., $q \geq 0.5$), then spectrum leasing is generally not advantageous for the primary. Moreover, in this case, the largest primary throughput under spectrum leasing is obtained with ST being

$$\begin{aligned}\frac{E[\text{Packets}]}{E[S]} = {} & 2P_1 + P_2\left(1 + (1 - P_{\mathrm{out},11}) + (1 - P_{\mathrm{out},2s})\left(1 - P_{\mathrm{out},s2}\left(\frac{q}{b_2}\right)\right) + P_{\mathrm{out},2s}(1 - P_{\mathrm{out},22})\right)\\ & + P_3\left(1 + (1 - P_{\mathrm{out},22}) + (1 - P_{\mathrm{out},1s})\left(1 - P_{\mathrm{out},s1}\left(\frac{q}{b_1}\right)\right) + P_{\mathrm{out},1s}(1 - P_{\mathrm{out},11})\right)\\ & + P_4\left(P_{\mathrm{out},1s}(1 - P_{\mathrm{out},11}) + (1 - P_{\mathrm{out},2s})\left(1 - P_{\mathrm{out},s2}(\alpha_2)\right) + (1 - P_{\mathrm{out},1s})\left(1 - P_{\mathrm{out},s1}(\alpha_1)\right)\right.\\ & \left. + P_{\mathrm{out},2s}(1 - P_{\mathrm{out},22})\right)(1 + P_2 + P_3 + P_4)^{-1}\end{aligned} \tag{8}$$

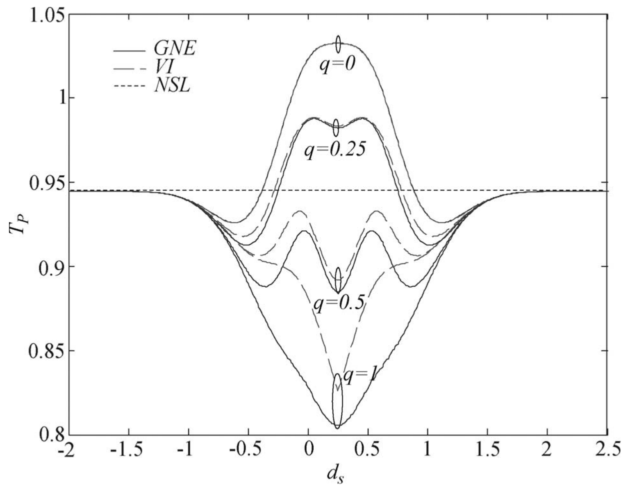

Fig. 4. Average primary system throughput T_P versus distance d_S for spectrum leasing based on GNE and VI solutions and for NSL for $(b_1, b_2) = (1, 1)$ $(R_1 = R_2 = 2, P = 1, \eta = 3)$.

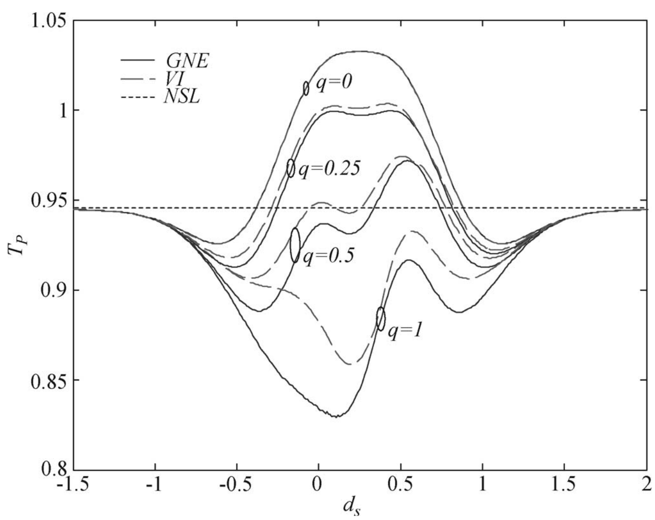

Fig. 5. Average primary system throughput T_P versus distance d_S for spectrum leasing based on GNE and VI solutions and for NSL for $(b_1, b_2) = (1, 2)$ $(R_1 = R_2 = 2, P = 1, \eta = 3)$.

closer to either PT_1 or PT_2, i.e., $d_s \simeq 0$ or $d_s \simeq 0.5$ since, otherwise, the benefits of cooperation are outweighed by the amount of spectrum leased. Note also that when ST is further away from PT_1 and PT_2, i.e., $d_s < -1.5$ or $d_s > 2$, decoding at ST is not possible, and the GNE and VI average throughput converge to that of NSL. Finally, it is noted that VI solutions perform better, as expected from the analysis, but the gains are not extremely large. This implies that, if complexity of signaling for calculation of (α_1, α_2) is an issue, one should resort to GNE solutions.

Fig. 5 plots the average primary system throughput T_P versus d_S for GNE and VI solutions with the same parameters as in the previous plot, with the difference that we set $b_1 = 1$ and $b_2 = 2$. In other words, in this setting, the spectrum leased by PT_2 is worth double to ST. As discussed, this could be the case if the channel quality experienced by the secondary user is higher on the spectral resource of PT_2. It can be shown that, in this setup, even for large QoS q, e.g., $q = 0.5$, as long as ST is sufficiently close to PT_2, e.g., $d_s \simeq 0.5$, spectrum leasing is still advantageous with respect to NSL. This is because, when ST is close to PT_2, the opportunity for PT_2 to lease a spectrum will be more frequent, and spectrum leasing by PT_2 is more efficient, as discussed.

V. Concluding Remarks

In this paper, we have extended the framework of spectrum leasing via cooperation by accounting for the presence of multiple primary users. With spectrum leasing via cooperation, secondary users gain access to the channel by cooperating with the primary users but under the QoS constraint that they receive enough spectral resources for transmission of their own data. The approach proposed in this paper is based on the observation that such QoS secondary requirement imposes a shared constraints on the spectrum-leasing decisions of the primary users. This is under the reasonable assumption that secondary users are interested in the overall amount of spectral resources they receive. The spectrum-leasing problem to be solved at the primary users is then formulated as a GNE problem, which, unlike conventional strategic games, enables one to impose a shared constraint on the players' actions. We have studied two classes of solutions that have different signaling requirements. An application of the framework that includes retransmissions has been also studied, leading to insight into the performance of spectrum leasing and of the proposed solutions.

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