# Spiking Neural Networks for Low-Power Edge Intelligence

Osvaldo Simeone Joint work with Hyeryung Jang (KCL), Bipin Rajendran (NJIT), Brian Gardner (USurrey), and André Grüning (HStralsund)

King's College London

University of Luxembourg, 18/9/2019





#### Overview

- Motivation and Introduction
- Applications
- Models
- Learning Algorithms
- Examples

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# Machine Learning Today



[Rajendran '18]

 Breakthroughs in ML using (deep) Artificial Neural Networks (ANNs) have come at the expense of massive memory, energy, and time requirements...

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# Machine Learning Today

Artificial Intelligence / Machine Learning

MIT Technology Review

#### Training a single AI model can emit as much carbon as five cars in their lifetimes

#### Common carbon footprint benchmarks

in lbs of CO2 equivalent



Chart: MIT Technology Review · Source: Strubell et al. · Created with Datawrapper

 Breakthroughs in ML using (deep) Artificial Neural Networks (ANNs) have come at the expense of massive memory, energy, and time requirements...

#### Resource-Constrained Machine Learning

 How to implement ML (inference and learning) on mobile or embedded devices with limited energy and memory resources? [Welling '18]



mobile personal assistants



IoT mobile or embedded devices



medical and health wearables



neural prosthetics

# Machine Learning at the Edge

- A solution is mobile edge or cloud computing: offload computations to an edge or cloud server.
- Possible privacy and latency issues



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- Possible privacy and latency issues



World Class Standards

# Machine Learning on Mobile Devices

- Another solution is to scale down energy and memory requirements of ANNs via tailored hardware implementations for mobile devices.
- Active field with established players and start-ups
- Trade-offs between accuracy and complexity
- Limited to inference



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# Neuromorphic Computing

• Spiking Neural Networks (SNNs) aim at attaining the efficiency of the human brain...







 $\sim$  20 Watts 2 sq. ft. & 1.4 Kg  $\sim 10^{15} \text{ ops/J}$ 

#### Neuromorphic Computing

- ... by taking inspiration from the dynamic, sparse, and event-driven learning and inference operation of the human brain.
- Neurons in the brain sense, process, and communicate over time using sparse binary signals (spikes or action potentials).



#### Spiking Neural Networks

- SNNs are networks of spiking neurons [Maas '97].
- Mostly studied in theoretical neuroscience, but recent interest from machine learning and hardware design researchers...



# Spiking Neural Networks

- Proof-of-concept and commercial hardware implementations of SNNs have demonstrated significant energy savings as compared to ANNs [Rajendran et al '19].
- $\bullet$  Energy consumed only when spikes are produced (binary values  $\rightarrow$  pJ/spike)



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Neuromorphic Computing

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## Spiking Neural Networks

• Increasing press coverage and positive market predictions...

#### WIRED

# The future of AI is neuromorphic. Meet the scientists building digital 'brains' for your phone

Neuromorphic chips are being designed to specifically mimic the human brain – and they could soon replace CPUs

Self-Learning Neuromorphic Chip Market Research Report- Global Forecast 2023



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 iniVation's Dynamic Vision Sensor (DVS), iniVation and AiCTX's Speck, Prophesee



• From IBM's DVS128 Gesture Dataset...



- Conversion from natural signals to spike signals theory [Lazar '06]
- Practice: Rate encoding, time encoding, population encoding; rate decoding, first-to-spike decoding,...



SNNs can perform

► Inference/ control: classification, regression, prediction, ...



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- SNNs can perform
  - ▶ Inference/ control: classification, regression, prediction, ...
  - Learning: supervised, unsupervised, reinforced



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- Data presentation through I/O interfaces:
  - Frame-based (or batch)

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- Data presentation through I/O interfaces:
  - Frame-based (or batch)
  - Streaming (or online)

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#### **SNN** Models

- SNNs are networks of spiking neurons.
- Their operation is defined by:
  - topology (connectivity)
  - neuron model



# Topology

- Arbitrary directed graph with directed links representing synaptic connections
- "Parent", or pre-synaptic, neuron affects causally spiking behavior of "child", or post-synaptic, neuron
- Enables recurrent connectivity (directed loops), including self-loops



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#### Topology

- Discrete (algorithmic) time, as in many practical implementations (e.g., Intel's Loihi)
- Binary outputs: 0 = no spike; and 1 = spike (energy consumed)



#### Neuron Model

- Each neuron is characterized by an internal state known as membrane potential [Gerstner and Kistler '02].
- Generally, a higher membrane potentially entails a larger propensity for spiking.
- The membrane potential evolves over time as a function of the past behavior of pre-synaptic neurons and of the neuron itself.

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#### Membrane Potential

$$u_{i,t} = \sum_{j \in \mathcal{P}_i} w_{j,i} (a_t * s_{j,t}) + w_i (b_t * s_{i,t}) + \gamma_i$$



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• Feedforward filter (kernel)  $a_t$  with learnable synaptic weight  $w_{j,i}$ 



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Feedback filter (kernel) b<sub>t</sub> with learnable w<sub>i</sub> (e.g., refractory period)
Bias (threshold) γ<sub>i</sub>


#### Membrane Potential

- Kernels can more generally be parameterized via multiple basis functions and learnable weights [Pillow et al '08].
- This allows learning of the temporal "receptive fields" of the neurons, e.g., by adapting synaptic delays.



• The most common model is leaky integrate-and-fire (LIF) [Gerstner and Kistler '02]: Spike when membrane potential is positive



- LIF-based SNNs can approximate operation of feedforward and recurrent ANNs: spiking rates in SNN  $\approx$  neuron outputs in ANN
- Often used for inference by converting a pre-trained ANN [Rueckauer et al '17]
- Direct training is required to leverage temporal information processing and learning
- This is made difficult by output non-differentiability with respect to model parameters
- Heuristic training algorithms based on approximations, such as surrogate gradient [Neftci '18] [Anwani and Rajendran '18]
- As for ANNs, these require backpropagation

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### Probabilistic SNN Models

 Probabilistic Generalized Linear Model (GLM): Conditional spiking probability [Pillow et al '08]

$$p(s_{i,t}=1|\mathbf{s}_{\leq t-1})=\sigma(u_{i,t})$$



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# Probabilistic SNN Models

- GLM SNN models are dynamic generalization of belief networks [Neal '92].
- They enable direct training of spike-based statistical criteria.

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# Training SNNs

- SNNs can be trained using supervised, unsupervised, and reinforcement learning.
- To fix the ideas, we focus here on supervised learning:

training data = {(input, output)}  $\rightarrow$  generalization

# Training SNNs

- Visible neurons, clamped to input/ output data
- Latent neurons, free



- Probabilistic SNNs can be directly trained using standard statistical criteria, such as maximum likelihood, maximum mutual information, or maximum average return.
- Maximum likelihood for supervised (and unsupervised) learning:

 $\max E_{data}[\ln p(visible)] = E_{data}[\ln E_{latent}p(visible, latent)]$ 

 Regularization terms are typically added, e.g., to limit spiking rate (and hence energy consumption) – bounded rationality [Leibfried and Braun '15]

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#### • Using Stochastic Gradient Descent (SGD) along with...

• ... variational inference to address marginalization over latent neurons yields the on-line learning rule [Rezende et al '11] [Osogami '17] [Jang et al '18]

 $w_{j,i} \leftarrow w_{j,i} + \eta \times \ell \times \text{pre-syn}_j \times \text{post-syn}_i$ 

- The learning signal  $\ell$  is a global feedback signal (akin to neuromodulator in neuroscience).
- Pre-synaptic and post-synaptic terms are local to each synapse.

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• Stochastic Gradient Descent (SGD) updates for synaptic weights:

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The learning signal ℓ is derived using variational inference [Rezende et al '11] [Osogami '17] [Jang et al '18]:

 $\ell_t = 1$  for visible (input and output) neurons,

$$\ell_t = \sum_{i \in \text{visible}} \log p(s_{i,t} | u_{i,t}) \text{ for latent neurons}$$

Positive feedback to hidden neurons if desired behavior has large probability

• Stochastic Gradient Descent (SGD) updates for synaptic weights:

 $w_{j,i} \leftarrow w_{j,i} + \eta \times \ell \times \text{pre-syn}_j \times \text{post-syn}_i$ 

- $\operatorname{pre-syn}_{i} = a_t * s_{j,t} = \operatorname{pre-synaptic trace}$
- Large if previous behavior of pre-synaptic neuron is consistent with synaptic receptive field (e.g., if recent spiking)

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$$w_{j,i} \leftarrow w_{j,i} + \eta \times \ell \times \text{pre-syn}_j \times \text{post-syn}_i$$

• 
$$\mathsf{post-syn}_i = s_{i,t} - \sigma(u_{i,t}) = \mathsf{post-synaptic}$$
 error

- Post-synaptic error = desired/ observed behavior model averaged behavior [Bienenstock et al '82]
- No backpropagation

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- Further simplifications yield Hebbian learning: "Neurons that fire together, wire together" [Hebb '49].



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# Batch Training



Spiking Neural Network (SNN)

# Batch Training

• Rate encoding/ decoding



# Batch Training

• Graceful trade-off complexity/ delay vs accuracy [Jang et al '18-2]



- Online training for prediction
- Fully connected (recurrent) SNN topology with hidden neurons



• Rate encoding



• Rate encoding



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Neuromorphic Computing

• Rate encoding



• Time encoding



• Rate vs time encoding



# **Concluding Remarks**

- Statistical signal processing review of neuromorphic computing via Spiking Neural Networks
- Additional topics:
  - more general energy-based probabilistic SNN models [Osogami '17] [Jang '19]
  - recurrent SNNs for long-term memory [Maas '11]
  - neural sampling: information encoded in steady-state behavior [Buesing et al '11]
  - Bayesian learning via Langevin dynamics [Pecevski et al '11] [Kappel et al '15]
- Some open problems:
  - ▶ meta-learning, life-long learning, transfer learning [Bellec et al '18]
  - training I/O interfaces [Lazar and Toth '03]
  - integration of ANNs and SNNs [Pei et al '19]

#### For More...

• H. Jang, O. Simeone, B. Gardner and A. Gruning, "An Introduction to Probabilistic Spiking Neural Networks," to appear on IEEE Signal Processing Magazine (available on arxiv).
### Acknowledgements

This work has received funding from the European Research Council (ERC) under the European Union's Horizon 2020 research and innovation programme (grant agreement No. 725731) and from the US National Science Foundation (NSF) under grant ECCS 1710009.

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## Reinforcement Learning



### Reinforcement Learning

• From [Rosenfeld et al '18]



## Applications

• Example: Keyword spotting based on audio streaming; accuracy comparable to ANN.



 With latent variables z, Maximum Likelihood requires the maximization of

$$\begin{split} \max_{\theta} & \ln p(x|\theta) = \ln \left( \sum_{z} p(x, z|\theta) \right) \\ &= \ln \left( \mathbb{E}_{z \sim p(z|\theta)} [p(z|x, \theta)] \right) \end{split}$$

• Key issue: Need to marginalize over latent variables, whose distribution is to be learned.

- VI tackles this problem by substituting the expectation for an optimization over a predictive, or variational, distribution q(z|x, φ)
- Optimization over both model parameters θ and variational parameters φ is done by maximizing the Evidence Lower BOund (ELBO) or (negative) free energy

$$\mathcal{L}(\theta,\varphi) = \mathbb{E}_{z \sim q(z|x,\varphi)} [\ln p(x,z|\theta) - \ln q(z|x,\varphi)]$$

learning signal  $\ell(x, z | \theta, \varphi)$ 

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• The ELBO is a global lower bound on the log-likelihood (LL) function

 $\ln p(x|\theta) \geq \mathcal{L}(\theta,\varphi),$ 

 Equality holds at a value θ<sub>0</sub> if and only if the distribution q(z|x, φ) is the posterior of z given x (i.e., optimal Bayesian estimate)

$$q(z|x,\varphi) = p(z|x,\theta_0).$$



• VI approximates the EM algorithm by using SGD over both variables with gradients

$$abla_{ heta} L( heta, arphi) pprox 
abla_{ heta} \log p(x, z),$$
 $abla_{arphi} L( heta, arphi) pprox \ell(x, z | heta, arphi) \cdot 
abla_{arphi} \log q(z | x, arphi)$ 
with  $z \sim q(z | x, arphi)$ 

Variational Inference (VI) for SNNs

• In order to simplify the evaluation of the learning signal and the learning rule, a typical choice for GLM SNNs is

$$q(z_{i,t}|x_{\leq T}, z_{\leq t-1}) = p(z_{i,t}|u_{i,t}, \theta)$$

• This leads to a simplified learning signal

$$\ell_t = \sum_i \log p(x_{i,t}|u_{i,t})$$

where the sum is over the observed spike trains.

• The learning signal measure the likelihood of the observed spike trains

### Pseudocode for Online Learning

• For each time t

- Sampling: Each hidden neuron *i* emits a spike with probability  $\sigma(u_{i,t})$
- ► Global feedback (1): A central processor updates the learning signal

$$\ell_t = \kappa \ell_{t-1} + (1-\kappa) \sum_i \log p(x_{i,t}|u_{i,t}),$$

where the sum is over the observed neurons, which is fed back to all latent neurons

- Global feedback (2): The central processor feeds back to all neurons any reward signal r<sub>t</sub> (if reinforcement learning)
- ► Local parameter update: Three-factor rule with  $M_t = \ell_t \times r_t$  for hidden neurons and  $M_t = r_t$  for the observed neurons