# PGTC 2022

Titles and Abstracts

## 1 Plenary Speakers

**Dr Alison Parker**
*University of Leeds*
Title: An introduction to diagram algebras
Abstract: This talk will give a brief introduction to several important diagram algebras. While there is no formal definition of a diagram algebra there are many examples, such as Braid groups, the Brauer algebra, the Temperley-Lieb algebra, the KLR algebra and other variations. I give an overview of some of the features of these algebras, including cellularity, cell modules, and how to determine when an algebra is semi-simple. I will also talk about some of the open problems in this area.

**Professor Cheryl Praeger**
*University of Western Australia*
Title: Cycles in Permutation Groups
Abstract: The question which I hope to address in this lecture is: Why care about cycles in permutation groups? It has engaged mathematicians for around 150 years, going back to Jordan's seminal result that the finite primitive permutation groups containing a prime length cycle with at least three fixed points are the giants, the alternating group and symmetric group. From the 1970's, this and other old results offered a means of identifying these giants among primitive groups computationally, using random selections to find such elements. The computational application

raised a further question: Just how easy is it to find, or construct, one of these Jordan cycles? I'll trace this story up to 2021, when with Stephen Glasby and Bill Unger we found a (to us surprising) answer. If there is time I'll draw a parallel with stingray matrices in finite classical groups.

**Professor Peter Cameron**
*University of St Andrews and Queen Mary University*
Title: Graphs on groups
Abstract: The title indicates graphs whose vertex set is a finite group (or something related, such as the set of conjugacy classes in a finite group) and whose edges reflect the group structure. The exemplar is the commuting graph, introduced for simple groups by Brauer and Fowler in a celebrated paper: the vertices are the non identity elements, two vertices joined if they commute.

I will give a sample of the many results which have been found in this area, including a strengthening of an old theorem of Landau, a new constant related to the clique number of power graphs of groups, and a method for finding potentially interesting graphs from finite simple groups (currently at the experimental stage), together with a number of open problems.

# 2 Contributed Talks

**Dean Yates**

*Queen Mary, University of London*

Title: Spin representations of the symmetric group

Abstract: We will compare the development of the projective representation theory of the symmetric group with the linear representation theory by considering methods for determining decomposition numbers for spin representations of the symmetric group over fields of positive characteristic; in particular, by constructing spin representations corresponding to partitions of n into 2 distinct parts via induction of the basic spin representation, indexed by the partition (n).

**Alfred Dabson**

*City, University of London*

Title: Blocks of Symmetric and General Linear Groups of Weight Two

Abstract: Broué's Abelian Defect Group Conjecture is known to be true for blocks of symmetric groups and unipotent blocks of general linear groups. Craven and Rouquier have conjectured that the derived equivalence in the latter case can be chosen to be perverse. A similar conjecture is investigated for the symmetric group blocks of weight two, with mind to how one may influence the other.

**Diego Martin Duro**

*University of Warwick*

Title: From the Knutson – Savitskii Conjecture to the Knutson Index of characters

Abstract: About 40 years ago, Knutson conjectured that for every irreducible character, there is a generalised character such that their tensor product is the regular character. Savitskii disproved this in 1993 and posed a new conjecture. In this talk, after a brief introduction to Character Theory, we will disprove Savitskii's Conjecture and discuss its relations to Kaplansky's Sixth Conjecture.

**Aura-Cristiana Radu**
*Newcastle University*
Title: Lifting modules for Chevalley Groups to Algebraic Groups
Abstract: Lifting $G(q)$-modules (joint work with Dr David Stewart) Let $G$ be a simple, simply connected algebraic group over an algebraically closed field $k$ of characteristic $p > 0$. Then, for the $r^{th}$ Frobenius map $F^r : G \to G$, the fixed point set of the points, $G(q) := G(k)^{F^r}$, is a finite group of Chevalley type. Let $L_1, \ldots, L_m$ be a list of not necessarily distinct simple $G$-modules which are $G(q)$-simple on restriction. Then by $L_1/L_2 \ldots /L_m$ we denote the finite length composition series of a finite-length module $M$. We explicitly determine the bounds on $r$ such that $G(q)$-modules with an arbitrary number of composition factors lift to the ambient algebraic group $G$. That is, we determine $r_0(p, \Phi, m)$, so that for any $r \geq r_0$, any $G(q)$-module $M$ of the form $L_1/ \ldots /L_m$ is the restriction of a $G$-module $\tilde{M}$ of the form $L_1/ \ldots /L_m$.

**Pavel Turek**
*Royal Holloway, University of London*
Title:Plethysms of the natural $KSL_2(\mathbb{F}_p)$-module in characteristic $p$
Abstract: The Schur functors are a key object in the study of the polynomial representations of the group $GL_n(K)$ over a field $K$. The composition of two Schur functors is called a plethysm and it is an ongoing question to find a simple combinatorial description of plethysms of the natural $KGL_n(K)$-module even when $K$ has characteristic zero. However, in particular cases some results are known. The aim of this talk is to look at "small" plethysms of the natural $KSL_2(\mathbb{F}_p)$-module when $K$ has characteristic $p$ and explore some ideas originally introduced by Almkvist, in particular the introduction of the cyclotomic fields into the question. Towards the end we should be able to use a result regarding cyclotomic units to classify all these plethysms which have precisely one non-projective summand.

**David Brown**

*University of Kent*

Title: Hemmer Nakano and the Ramified Partition Algebra

Abstract: We will discuss the important Hemmer Nakano theorem on Specht filtration multiplicities for the symmetric group, moving on to the corresponding result for its older cousin the partition algebra. We will then motivate and introduce the ramified partition algebra, and give some idea of how a corresponding result takes shape in this algebra.

**Precious Ugonwanyi Agigor-Mike**

*Birkbeck University of London*

Title: Triple Product Free Sets

Abstract: Let $S$ be a non-empty subset of finite group $G$. Then we say $S$ is *triple product free* if for all $a, b, c \in S$, $abc \notin S$.

That is to say that $S \cap SSS = \emptyset$ , where $SSS = \{abc | a, b, c \in S\}$. Furthermore, $S$ is said to be locally maximal triple product free in $G$ if $S$ is triple product free and not properly contained in any other triple product free set. We give some examples and background, before describing some results on groups containing small locally maximal triple product free sets.

**Aluna Rizzoli**

*University of Cambridge*

Title: On the isometry group of a finite dimensional Banach space

Abstract: For a closed subgroup $H$ of $\mathrm{GL}_n(\mathbb{R})$, define $\hat{H}$ to be the largest subgroup of $\mathrm{GL}_n(\mathbb{R})$ that has the same orbits as $H$ on $\mathbb{R}^n$. We prove that $H$ is the full isometry group of a norm on $\mathbb{R}^n$ if and only if $\pm I \in H$ and $H = \hat{H}$. Using this, we show that every compact Lie group with a central involution can be realised as the isometry group of a norm. We then study the relationship between $H$ and $\hat{H}$ for compact $H \leq \mathrm{GL}_n(\mathbb{R})$, showing that, with specified exceptions, $H$ and $\hat{H}$ have the same connected component of the identity. This is joint work with Emmanuel Breuillard, Martin Liebeck and Assaf Naor.

**James Timmins**

*University of Oxford*

Title: Measuring representations of $p$-adic groups

Abstract: In this talk I'll introduce $p$-adic groups and their (mod p) representations. These representations can be understood as finitely-generated modules over a generalisation of the group algebra, known as the Iwasawa algebra. I'll present questions about the "size" of these algebras and their modules, and outline recent progress towards answers.

**Gemma Crowe**

*Heriot-Watt University*

Title: Conjugacy in extensions of right-angled Artin groups.

Abstract: The conjugacy problem in RAAGs has been known for over 30 years, and properties including conjugacy growth and representatives from a formal language point of view have been well-studied in RAAGs and similar groups. In this talk I will discuss some of these results and important properties of RAAGs, before discussing some of my own work which has focused on what happens when we try to analyse similar problems for virtual RAAGs.

**Georges Neaime**

*Ruhr University Bochum (Germany)*

Title: Interval groups of finite Coxeter groups

Abstract: We study the interval groups associated with the intervals of quasi-Coxeter elements in a finite Coxeter group. We determine when these interval groups are Garside groups. We also describe them by presentations with generators and relations by using Carter admissible diagrams. Finally, we relate our study to the theory of Artin-Tits groups.

**Lewis Molyneux**
*Royal Holloway, University of London*
Title: An Irrational Slope Thompson Group and its BNSR-Invariant
Abstract: In 2000, Cleary wrote of a Thompson Group variant with irrational slopes and breakpoints. More recently, an explicit finite presentation for this group has been found, along with a number of surprising properties that distinguish these irrational slope variants from the typical Thompson Group variants. In this talk, we will discuss the basics of Thompson Groups, what is different about those with irrational slopes, along with a discussion of the BNSR-invariant and its behaviour for Thompson Group variants, culminating in a demonstrated calculation of the BNSR-invariant for F, the first known irrational slope Thompson Group.

**Antoine Goldsborough**
*Heriot-Watt University*
Title : random walks and quasi-isometries
Abstract : The study of random walks on various groups is a very rich area of mathematics and a considerable amount of research has been done on these. In geometric group theory, a crucial notion is that of a "quasi-isometry" between two spaces. Sadly, random walks and quasi-isometries do not behave well together. In this talk, we will propose the study of a more general process, namely of a Markov chain in order to resolve this issue. This leads to interesting results about random walks on groups quasi-isometric to specific groups, including a Central Limit Theorem. This is joint work with Alessandro Sisto.

**Rachel Pengelly**
*University of Birmingham*
Title: Exceptional $sl_2$-triples
Abstract: Given three elements of a Lie algebra over an algebraically closed field K, we say that these elements form an $sl_2$-triple if they generate a subalgebra which is a homomorphic image of $sl_2(K)$. In this talk I will discuss $sl_2$-triples in the exceptional Lie algebras over fields of prime characteristic. In particular, I will explore the connection between G-complete reducibility and the existence of a bijection between the orbits of nilpotent elements of the Lie algebra and the orbits of $sl_2$-triples.

**Jamie Mason**
*University of Birmingham*
Title: Automorphism of Objective Partial Groups
Abstract: Partial groups were introduced by Chermak in 2013 and are closely related to the theory of fusion systems and therefore may help in Ashbacker's proof of the classification of finite simple groups. Since then a general theory of partial groups has begun to crystallise. I will introduce some of this theory and then go on to talk about automorphism groups of partial groups. Then I will finish with a discussion of whether the category of objective partial groups is universal.

**Damiano Rossi**
*City, University of London*
Title: Nilpotent blocks and isomorphisms of character triples
Abstract: Dade's Projective Conjecture (DPC) provides a way of unifying the so-called counting conjectures in representation theory of finite groups into a single statement. In order to prove this statement via an application of the classification of finite simple groups, we need to control the interaction between the representations of a finite group and those of its simple factors. This is done by using Clifford theory. In 2017, Spaeth introduced a significant strengthening of DPC compatible with Clifford theory, known as the Character Triple Conjecture (CTC). Moreover she proved that if the CTC holds for every quasisimple group, then DPC holds for every finite group. Nonetheless, the CTC is expected to hold for every finite group and it has been shown to hold for the class of p-solvable groups. In this talk, we show that the CTC holds for every block covering a nilpotent block.

**Emily Hall**
*University of Bristol*
Title: Almost Elusive Groups
Abstract: Let G be a transitive permutation group on a finite set X such that |X|>1.We say an element of G is a derangement if it has no fixed points on X. As a consequence of the orbit-counting lemma we know that such elements always exist in G, but what if we seek derangements with special properties? For example, do derangements of prime order always exist? In this talk I will discuss this question and use it to motivate the concept of almost elusive groups. I will also give a brief description of the classification of these almost elusive groups in the primitive setting and discuss some of the key concepts behind the proof.

**Hongyi Huang**
*University of Bristol*
Title: Base-two primitive permutation groups
Abstract: Let $G \leqslant \mathrm{Sym}(\Omega)$ be a permutation group and recall that a base for $G$ is a subset of $\Omega$ with trivial pointwise stabiliser. Bases have been studied for many decades and an ambitious project initiated by Jan Saxl in the 1990s seeks to determine the primitive groups with a base of size 2. In recent years, there has been significant progress towards this goal and I will review some of the latest developments. I will also report on recent work concerning the Saxl graph of a base-two permutation group, which was recently introduced by Burness and Giudici. Some of the new results are based on recent joint work with Tim Burness.