

Title: Graduated orders and their lattices.

Abstract: For  $G$  a finite group,  $p$  a prime and  $(K, \mathcal{O}_K, k)$  a  $p$ -modular system the group ring  $\mathcal{O}_K G$  is an  $\mathcal{O}_k$ -order in the  $K$ -algebra  $KG$ . Graduated  $\mathcal{O}_K$ -orders are a particularly nice class of  $\mathcal{O}_K$ -orders first introduced by Zassenhaus. In this talk will see that an  $\mathcal{O}_K$ -order  $\Lambda$  in a split  $K$ -algebra  $A$  is graduated if the decomposition numbers for the regular  $A$ -module are no greater than 1. Furthermore will see that graduated orders can be described (not uniquely) by a tuple  $n$  and a matrix  $M$  called the exponent matrix. Finding a suitable  $n$  and  $M$  for a graduated order  $\Lambda$  in the  $K$ -algebra  $A$  provides a parameterisation of the  $\Lambda$ -lattices inside the regular  $A$ -module. Understanding the  $\mathcal{O}_K G$ -lattices inside representations of certain groups  $G$  is of interest to those involved in the Langlands programme as well as of independent interest to algebraists.