A very brief introduction to modular representation theory.

For K a number field it is natural to look at the representations of the Galois group of a finite extension not only over K but also over its ring of integers. Taking reduction modulo a prime ideal of these integral representations we get examples of representations over fields of finite characteristic naturally occurring in number theory. For G a finite group and K a field of characteristic 0, Mashke's theorem says that every finite dimensional KG-module is semisimple. This is true also if K is a field of characteristic p, where p is a prime not dividing the order of G. However if K is a field of prime characteristic p that divides the order of G then Mashke's theorem no longer holds and the building blocks of KG-modules can no longer be assumed to be simple. The study of such KG-modules is called modular representation theory and in this talk we will give a whistle stop tour of some of its key introductory results.