

Title: The diameter of the modular McKay graph of $SL_n(\mathbb{F}_p)$.

Abstract: For G a finite group, F an algebraically closed field and W a faithful FG -module the McKay graph, $\mathcal{M}_F(G, W)$ is a directed graph on the set of simple FG -modules. There is an edge in the graph from V_1 to V_2 if V_2 occurs as a composition factor of $V_1 \otimes W$. These graphs famously come up in the McKay correspondence which says that such graphs for the group $SU_2(\mathbb{C})$ are affine Dynkin diagrams of type A, D or E .

In the case where the characteristic of F divides the order of G , finding the composition factors of tensor products is a hard problem. However it might surprise you to know that taking G to be $SL_n(\mathbb{F}_p)$ and W the standard n -dimensional $\overline{\mathbb{F}}_p SL_n(\mathbb{F}_p)$ -module we can show

$$\text{diam} \mathcal{M}_{\overline{\mathbb{F}}_p}(G, W) = \frac{1}{2}(p-1)(n^2 - n).$$

We are able to prove this neat formula without explicitly constructing the graphs. This is of particular interest to people looking at mixing times for random walks on McKay graphs.