Workshop 8: Forward/Inverse Kinematics

1 Getting Started

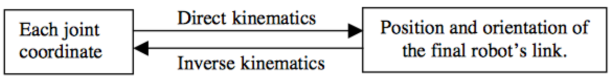
Make sure you have everything you need to complete this lab:

- a LEGO Mindstorms EV3 robot with two motors
- a computer with LEGO Mindstorms EV3 software application installed
- a USB cable that connects the robot to your computer
- a piece of square paper

2 Introduction to the mathematics of kinematic modelling

2.1 Forward/Inverse Kinematics

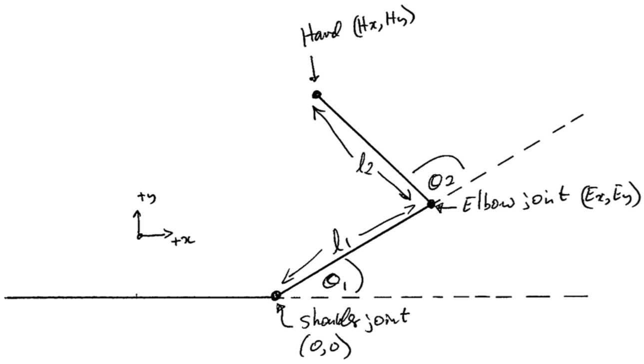
In order to adjust the robot's motor or arm by a specific angle or to calculate the position of the robot's final link (arm), **kinematic mathematical** model is needed. The kinematics is the study of the **robot's movements** with regard to a reference system or in other words it's a **relationship**, between the **position** and the orientation of the **robot's final link and the values of their joint coordinates**. Here is an image below, explaining the difference between **Forward (Direct) and Inverse Kinematics:**



So as you can see, **Forward Kinematics** is having each joint coordinates and calculate the **position and orientation** of each joint at the **final robot's link**. While **Inverse Kinematics** is having the position and the orientation of the final robot link and calculate each joint coordinates from it.
The equation to calculate the position of the robot's final link in the coordinate system:

$$r = \begin{pmatrix} r_1 \\ r_2 \end{pmatrix} = \begin{pmatrix} l_1 * cos\theta_1 + l_2 * cos(\theta_1 + \theta_2) \\ l_1 * sin\theta_1 + l_2 * sin(\theta_1 + \theta_2) \end{pmatrix}$$

where **r** is the final position of the robot's arm located at r_1, r_2 in the coordinate system. l_1 and r_2 are the length of the arms respectively and **theta** is the joint angle. The image below shows how the equation works, and helps you to understand the calculation:



2.2 Differentiation Kinematics

The other important and also interesting part of kinematics is **Differentiation Kinematics**. You can also calculate the angular changes during the arm movement. For this equation you will need to use differentiation and 2x2 matrix inverse. Here is the following equations how to calculate the angular changes:

$$\Delta\Theta = I^{-1} * \Delta r$$

The matrix **I** is the following:

$$I = \begin{pmatrix} \frac{dr_1}{d\theta_1} & \frac{dr_1}{d\theta_2} \\ \frac{dr_2}{d\theta_1} & \frac{dr_2}{d\theta_2} \end{pmatrix}$$

And to calculate the inverse of a **2x2 matrix**, use the example below:

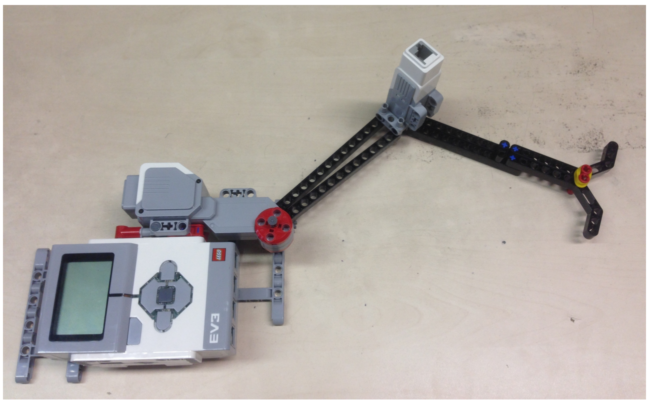
$$A = \begin{pmatrix} a & b \\ b & c \end{pmatrix} \quad A^{-1} = \frac{1}{a * d - b * c} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

At the end of the equation you should get a matrix multiplication, here is how the **matrix multiplication** works:

$$A_x = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} ax_1 + bx_2 \\ cx_1 + dx_2 \end{pmatrix}$$

3 Coordinated control arm

In this week's workshop you are going to build a simple coordinated robot arm, using two EV3 motors.



1. Follow the building instructions provided for the handout, and build the robot arm.
2. Once you have finished you should be able to move the robot arm by rotate the gears itself, without any additional programming.

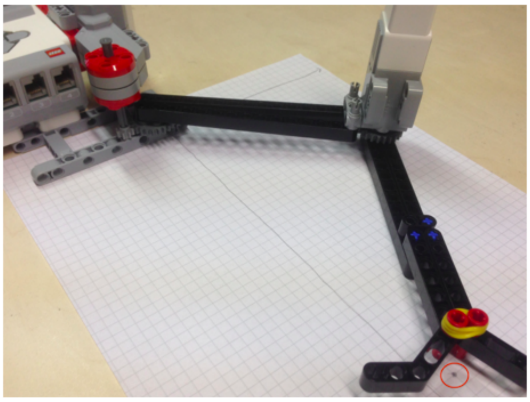
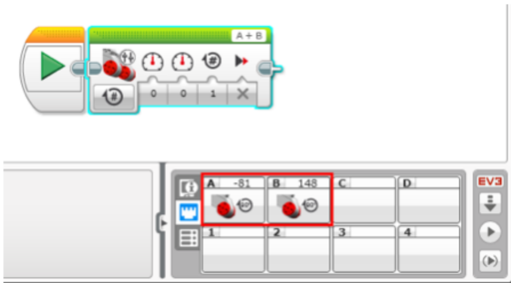
4 Calculate Forward/Inverse Kinematics

In this section we are going to perform a couple of **forward/inverse and differentiation kinematics** to make sure you understand the concept. You will need a piece of square paper to check your answers.

1. Place the arm to the paper horizontally. The arm's first joint should be positioned at the (0,0) coordinate in our coordinate system. **Measure** the **length of the first arm** until the second joint, then the **second part of the arm**.

2. Connect your robot arm to the EV3's brain and then connect the EV3 to your computer. **Create** a blank project with one **Large Motor** block, **choose** the appropriate port on the **Large Motor Block**.

3. Once you have done this, you should see two 0s in the Port View (bottom right corner). **Move** the arm manually to a random position and **record** the degree changes on each motor! This is going to be our $theta_1$ and $theta_2$. **Also record the coordinate of the robot's final link**:

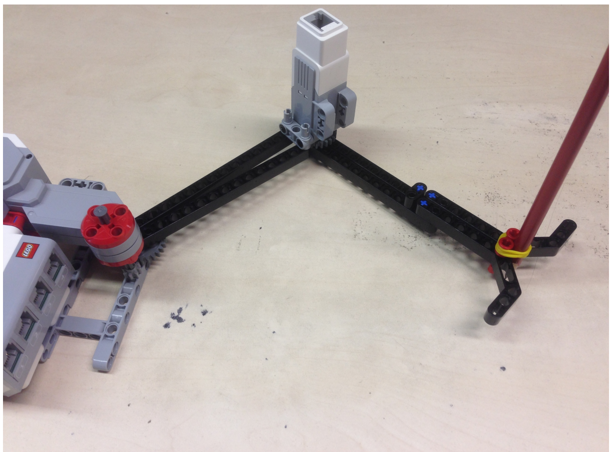
4. **Calculate** the forward kinematics of the robot arm by perform the equation above (see in section 2) and **determine the coordinate of the robot's final link**. Once you have the coordinate, compare it with the point you recorded on the paper and check if it's correct.

5. **Calculate** the differentiation kinematics for the same values. What is the overall angle changes ($d\Theta$)? Use the equations from **Section 2** for further help.

5 Draw with a pen attached

You will need to repeat the drawing exercises from Workshop 1, but now using **Forward Kinematics**.

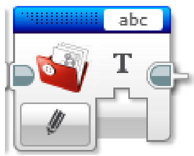
1. First, **build** a pen holder and attach it to the robot's final link. It should look like this:



2. Start with **drawing a circle**, then a **rectangle or a triangle**, just like you did in Workshop 1. Using kinematics should make the the drawing easier and also more accurate (no additional small circles at the corners).
3. Once you have finished the basic shape drawing. Draw a **five start**. Remember, the good thing in a five star is that you do not need to lift the pen, just need one line.

6 Challenge: Record and Repeat

In the exercises above you calculated forward/inverse kinematics and the angle changes of your robot arm during a movement. Now calculate the movements for the arm to **draw** some basic **shape with a pen attached**. **Record** all the angle changes of the two motors and save it. After the arm finished its movement, the robot should **repeat the same movement by itself**.



HINT: You might want to use the **File Access** block above, from one of the previous workshops to save the data into a file and read from it.

7 Challenge: Advanced Drawing

On the first week you managed to do some basic shape drawing by attaching a pen to a car. And now you have successfully did the same thing with **Forward Kinematics**. In this challenge you will need to repeat the first week's challenge, **write your name on a piece of paper** with the coordinated control arm, but now using **Forward Kinematics**.