AC Complement Problems: Satisfiability and Negation Elimination

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Abstract

In general, functional programs are assumed to be non-ambiguous. A simple way to handle ambiguous definitions is to replace them by non-ambiguous ones containing complement problems, which in turn have to be transformed into explicit expressions. Finding an explicit formulation for a complement problem is the same as eliminating negation from the corresponding equational formula. This problem was already shown decidable for complement problems interpreted in $T(\mathcal{F})$, and in $T(\mathcal{F})/_{=_E}$ when E is a permutative theory of Mal'cev. Here, we show that negation elimination is also decidable for linear complement problems interpreted in $T(\mathcal{F})/_{=_{AC}}$, where AC is a set of associative and commutative axioms. We present a system of rules that transforms any linear complement problem into simple formulae, and we give a decision test for simple formulae which serves as a basis for the development of a negation elimination algorithm. In the case of non-linear AC-complement problems even satisfiability is an open problem, but we will show, using the previous results, that satisfiability is decidable for a particular class of non-linear problems: when variables do not occur directly below AC-function symbols. Finally, we generalize the negation elimination results to a wider class of equational formulae modulo AC.

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