# Maude Summer School: Lecture 3-I

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These three properties have a straightforward generalization modulo axioms $B$, replacing the relation $\to_{\vec{E}}$ by the relation $\to_{\vec{E}/B}$.

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# Determinism = Confluence

Determinism is captured by confluence. The rules $\vec{E}$ of $(\Sigma, B, \vec{E})$ are confluent modulo $B$ iff for each $\Sigma$-term $t$, whenever $t \rightarrow^{\circledast}_{\vec{E}/B} u$ and $t \rightarrow^{\circledast}_{\vec{E}/B} v$, there is a $w$ such that $u \rightarrow^{\circledast}_{\vec{E}/B} w$ and $v \rightarrow^{\circledast}_{\vec{E}/B} w$.

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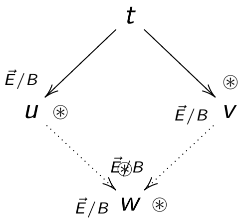

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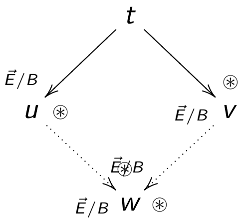


Under the termination assumption, confluence is decidable and checkable by Maude's Church-Rosser Checker (CRC) tool.

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A. Riesco, "MUnit: A Unit Framework for Maude," Proc. WRLA 2018, LNCS 11152, pp. 45–58, 2018.