Symbolic Computation in Maude: Some Tapas

José Meseguer

University of Illinois at Urbana-Champaign

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A complete set of variants of t can be computed as those t' such that $t \overset{\theta}{\rightsquigarrow}^{*}_{E,B} t'$ and t' is in E, B-normal form.

Folding variant narrowing is a strategy to compute a complete set of most general variants using variant subsumption.

$(\Sigma, E \cup B)$ has the finite variant property (FVP) iff folding variant narrowing terminates for any term t with a finite set of most general variants. FVP is semi-decidable and easily checkable in Maude when it holds.

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For example, $x > y \equiv y > x$ has the single unifier $\{x \mapsto y\}$ modulo $\mathcal{N}_{+,>}$.

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1. first solve the sytem of equations $head(l) > head(l') = \top \wedge head(l) > 1 + 1 + 1 = \bot$ modulo the composed theory. There are six constructor unifiers. The first is: $\alpha = \{l \mapsto (1 + 1 + 1); l_1, l' \mapsto (1 + 1); l_2\}$.
2. This shows that the formula is satisfiable, because $\{(1 + 1); nil\} \subseteq \{(1 + 1 + 1); l_1, (1 + 1); l_2, \emptyset\} \neq tt$, is irreducible by the equations for $\subseteq$ modulo ACU.

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Note that narrowing happens at two levels:

- with rules R modulo $E \cup B$ to perform symbolic transitions
- with E modulo B to compute $E \cup B$-unifiers.

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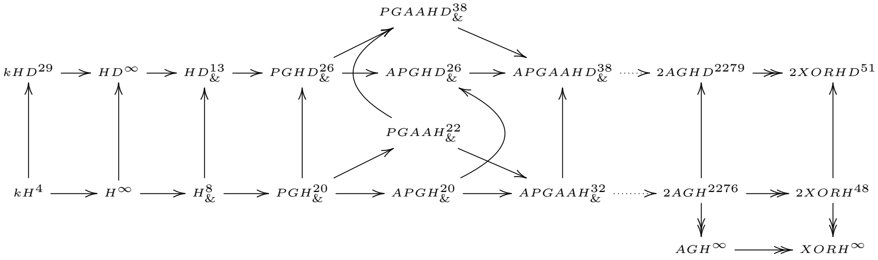

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