Titles and abstracts - Iwasawa 2015

Main talks

Ashay Burungale
Title: On the non-triviality of the p-adic height and p-adic Abel-Jacobi image
Abstract: Generalised Heegner cycles are associated to a pair of an elliptic Hecke eigenform and a Hecke character over an imaginary quadratic extension K. Let p be an odd prime split in K. We describe recent results on the non-triviality of the p-adic height and p-adic Abel-Jacobi image of generalised Heegner cycles over anti-cyclotomic extensions of K. The later non-triviality is rather general and holds in the non-CM case. The former non-triviality holds in the CM case and is a joint work in progress with D. Disegni.

Ted Chinburg
Title: Second Chern classes in Iwasawa theory
Abstract: Many of the Main Conjectures of Iwasawa theory relate the codimension one behavior of Iwasawa modules to p-adic L-functions. In this talk I will describe work with F. Bleher, R. Greenberg, M. Kakde, G. Pappas, R. Sharifi and M. Taylor on codimension two behavior when the modules in question are trivial in codimension one. Over imaginary quadratic fields, the second Chern classes of certain Iwasawa modules can be determined using symbols in K_2 groups constructed from Katz p-adic L-functions. By the end of the talk I will discuss a non-commutative version of this result.

Henri Darmon
Title: Stark-Heegner points and generalised Kato classes
Abstract: I will describe a formula relating the two objects in the title, and discuss its applications to the arithmetic of elliptic curves over ring class fields of real quadratic fields. This is joint work with Victor Rotger.

Samit Dasgupta
Title: On p-adic Stark conjectures for real quadratic fields
Abstract: We will recall the statements of various p-adic Stark conjectures for real quadratic fields, and pose a new conjecture for characters of mixed signature.

Mladen Dimitrov
Title: On the exceptional zeros of p-adic L-functions of Hilbert modular forms
Abstract: The use of modular symbols to attach p-adic L-functions to Hecke eigenforms goes back to the work of Manin et al in the 70s. In the 90s, Stevens developed his theory of overconvergent modular symbols, which was successfully used to construct p-adic L-functions on the eigenvariety. In this talk we will present a work in collaboration with Daniel Barrera and Andrei Jorza in which we generalise this approach to

the Hilbert modular setting with a view towards applications to the exceptional zero conjecture.

Tim Dokchitser
Title: Growth of Sha in towers for isogenous curves
Abstract: I will discuss the growth of Sha for isogenous abelian varieties in towers of number fields, with particular emphasis on elliptic curves in cyclotomic towers. This is related to the phenomenon of 'positive mu-invariant' that accounts for the exponential growth of the p-part of Sha in the cyclotomic p-tower. This is joint work with Vladimir Dokchitser.

Olivier Fouquet Title: The Equivariant Tamagawa Number Conjecture with coefficients in Hecke algebra(s).
Abstract: In 1979, Barry Mazur asked whether the special values of the L-functions of two eigencuspforms congruent modulo an ideal m of the Hecke algebra could be computed in terms of the action of the Hecke algebra on the m-torsion of the étale cohomology of the modular curve. The Equivariant Tamagawa Number Conjecture with coefficients in the Hecke algebra provides a subtle and powerful answer to this question for congruent motives occurring in the cohomology of Shimura varieties. I will explain the motivation behind this conjecture, its statement and its proof for $GL2/\mathbb{Q}$ under the hypotheses ensuring the existence of a Taylor-Wiles system.

Ming-Lun Hsieh
Title: Anticyclotomic Iwasawa theory for modular forms
Abstract: We will report the recent development of anticyclotomic Iwasawa theory for elliptic modular forms of higher weights. For example, we will discuss an analogue of Mazur's conjecture for modular forms.

Dohyeong Kim
Title: On the transfer congruence between p-adic Hecke L-functions
Abstract: The existence of a non-commutative p-adic L-function predicts a family of congruences between special values of L-functions. We will prove a simplest such congruence of non-abelian nature, namely the transfer congruence, in the case of p-adic Hecke L-functions. We use a mildly improved version of Hsieh's Eisenstein series, and deduce the desired congruence by applying the q-expansion principle.

Guido Kings
Title: Explicit reciprocity laws for Rankin convolutions
Abstract In our joint work with Loeffler and Zerbes an explicit reciprocity law for Rankin-convolutions of modular forms was proved. The strategy of proof relies on using non-critical points of the p-adic L-function and a theory of p-adic interpolation of the étale realisation of motivic Eisenstein classes.

In this talk we explain our approach, discuss other cases where this strategy works and describe a new result, which generalizes the p-adic interpolation of motivic Eisenstein classes.

Preda Mihailescu
Title: On new approaches to classical conjectures in Iwasawa theory

Andreas Nickel
Title: Hybrid Iwasawa algebras and the equivariant Iwasawa main conjecture
Abstract: We discuss how the understanding of the structure of Iwasawa algebras can be used to prove the equivariant Iwasawa main conjecture for totally real fields for an infinite class of one-dimensional non-abelian p-adic Lie extensions. Crucially, we will not assume the vanishing of Iwasawa's μ-invariant. If time permits, we will also discuss applications to the (non-abelian) Brumer-Stark conjecture. This is joint work with Henri Johnston.

Takamichi Sano
Title: Arithmetic properties of zeta elements
Abstract: (Joint work with D. Burns and M. Kurihara) The existence of zeta elements is predicted by the equivariant Tamagawa number conjecture. I will discuss some new arithmetic properties of zeta elements. I will also describe some Iwasawa theoretic aspects of our theory.

Chris Skinner
Title:
Abstract:

Ki-Seng Tan
Title: Iwasawa main conjecture for elliptic curves over global function fields **Abstract:** Consider a global function field $K = \mathbb{F}_q(\mathcal{X})$, $\mathcal{X}$ a complete smooth curve over $\mathbb{F}_q$. Let A be an ordinary elliptic curve defined over K. A formula of Mazur defines for each $\mathbb{F}_q$-rational effective divisor D on $\mathcal{X}$, an element Θ_D in the group ring of the Weil group W_D, which interpolates special values of L-functions associated to A/K and characters of W_D. Let L/K be a $\mathbb{Z}_p^d$-extension unramified outside a finite set S consisting of ordinary places of K. Denote $\Gamma := Gal(L/K)$ and $\Lambda := \mathbb{Z}_p[[\Gamma]]$. Let X_L denote the dual p-Selmer group of A/L and let $CH_\Lambda(X_L)$ denote the characteristic ideal. In this talk, we introduce a (modified) p-adic L-function $\mathcal{L}_{L/K}$ derived from those Θ_D, $\mathrm{Supp}D \subset S$, and conjecture that it generates $CH_\Lambda(X_L)$. We shall give some evidence of the conjecture.

Eric Urban
Title:
Abstract:

Vinayak Vatsal
Title: Families of modular forms of half integral weight.
Abstract: Suppose F and G are holomorphic cuspidal newsforms of even weight and trivial characters of levels M and N respectively, such that F and G are congruent modulo a prime P in the algebraic closure of $\mathbf{Q}$. We can then pose the question of whether or not the modular forms associated to F and G by the Shimura-Waldspurger correspondence are also congruent modulo P. One can also ask whether the P-adic families associated to F and G survive the Shimura correspondence.

In considering these questions, one quickly realizes that in the most naive form the answer is negative, but the reason for the failure turns out to be quite subtle. The main point is that the usual Shimura-Waldspurger correspondence does not even yield a canonical bijection on the level of automorphic representations, and existence of half-integral congruences and P-adic families depends strongly on the choice of representation in the Waldspurger packet. While the question of p-adic families has been investigated by numerous authors (Hida, Stevens, Ramsey), the results are regrettably incomplete. The question of congruences does not seem to have been studied, beyond some informal speculations (Prasanna).

Our investigation of these questions reveals that the natural normalization for modular forms of half integral weight leads to the appearance of certain Tamagawa factors related to L-invariants of the corresponding integral weight representation. In view of the proof due to Greenberg and Stevens that the L-invariant is the derivative of a p-adic L-function at $s = 1$, and the appearance in their proof of a 2-variable p-adic L-function, we are led to examine the properties of p-adic families of modular forms of half integral weight, and to investigate the presence of trivial zeroes and derivatives in these families. We carry out this procedure in the case of families of tame level 1.

The study relies on a detailed analysis of the structure of the Waldspurger packets on the metaplectic group, and on a careful determination of which of the many possible elements of the Waldspurger packet actually show up the families in constructed by Stevens.

Otmar Venjakob

Title: Local Iwasawa cohomology and (φ, Γ)-modules over Lubin-Tate extensions

Abstract: For the p-cyclotomic tower of $\mathbb{Q}_p$ one has Fontaine's description of local Iwasawa cohomology in terms of the ψ-operator attached to any étale (φ, Γ)-module. In this talk I will report on joint work with Peter Schneider which consists of generalising Fontaine's result to the case of arbitratry Lubin-Tate towers over finite extensions L of $\mathbb{Q}_p$. In particular, we prove a kind of explicit reciprocity law which calculates the related Kummer map using Coleman power series.

Stefano Vigni

Title: Plus/minus Heegner points and Iwasawa theory of supersingular elliptic curves

Abstract: Let E be a rational elliptic curve (without complex multiplication) and let p be a prime of good supersingular reduction for E. Let K be an imaginary quadratic field satisfying a modified "Heegner hypothesis" in which p splits. In this talk I will explain how one can prove that Kobayashi's plus/minus p-primary Selmer groups of E over the anticyclotomic $\mathbb{Z}_p$-extension of K have corank 1 over the corresponding Iwasawa algebra Λ. Our strategy is based on an extension to the supersingular case of the Λ-adic Kolyvagin method originally developed by Bertolini in the ordinary setting. Applications to the growth of Selmer groups will be given. This is joint work with Matteo Longo.

Poster Presentations

Carl Wang Erickson
Title: Pseudo-modularity and Iwasawa theory
Abstract: Hida theory says that any ordinary eigenform lies in a family of ordinary eigenforms with p-adically varying coefficients and weight. Sometimes, these families collide. We will present joint work with Preston Wake in which we investigate the collisions between the Eisenstein family and cuspidal families, showing that a mild condition on class groups (which is implied by Greenberg's conjecture) is equivalent to the condition that the collision is a plane singularity. We also determine when it is a simple normal crossing and draw consequences in Iwasawa theory, namely, new cases of Sharifi's conjecture. The technique is to construct a deformation ring for ordinary Galois pseudorepresentations and compare this ring with the local ring on the eigencurve; in fact, we show they are isomorphic, a "pseudo-modularity" theorem.

Minoru Hirose
Title: Refinement of Rubin-Stark Conjecture by Shintani zeta functions
Abstract: Let H/F be an abelian extension of a number field and S, T finite sets of places of F satisfying some conditions. Let $\zeta_{S,T}(\sigma, s)$ be a (p-adic or complex) partial zeta function for $\sigma \in \mathrm{Gal}(H/F)$. It is conjectured that the leading coefficient of the Taylor expansion of $\zeta_{S,T}(\sigma, s)$ at $s = 0$ is given by the regulator of Rubin-Stark element. Especially in the rank one case, the regulator is equal to the logarithm of the field norm of special unit of H. These special units are called Gross-Stark units. Dasgupta gives a conjectural formula for the Gross-Stark unit itself by using a Shintani zeta function. One of the important point of his formula is that it gives a Gross-Stark unit itself, not a logarithm of a field norm of a Gross-Stark unit. We generalize his conjectural formula to the higher rank case. We propose a refinement of Rubin-Stark conjecture by giving a conjectural formula for the Rubin-Stark elements.

Ignazio Longhi
Title: Iwasawa main conjecture for the Carlitz cyclotomic extension
Abstract: Let F denote the function field $\mathbb{F}_q(\theta)$ and A be its "ring of integers", $A := \mathbb{F}_q[\theta]$. Then one can use the Carlitz module to attach to any prime ideal $\mathfrak{p}$ in A a "cyclotomic" extension $\mathcal{F}/F$, with Galois group $\Delta \times \Gamma$, where $\Delta \cong (A/\mathfrak{p})^*$ is finite and $\Gamma \cong \mathbb{Z}_p^{\mathbb{N}}$. Moreover, as discovered by Carlitz and later developed by Goss, one can define a characteristic p zeta function $\zeta_A(s) := \sum_{a \in A_+} a^{-s}$ (where $A_+ \subset A$ is the set of monic polynomials and s varies in $\mathbb{C}_\infty^* \times \mathbb{Z}_p$, with $\mathbb{C}_\infty$ a completion of an algebraic closure of $\mathbb{F}_q[[\theta^{-1}]]$). Similarly to the Riemann zeta function, special values of ζ_A can be $\mathfrak{p}$-adically interpolated by a function $L_\mathfrak{p}(X, y, \omega_\mathfrak{p}^i) \in A_\mathfrak{p}[[X]]$.

In the paper *Iwasawa main conjecture for the Carlitz cyclotomic extension and applications* (B. Anglès, A. Bandini, F. Bars and I. Longhi), we prove an analogue of the Iwasawa Main Conjecture and the Ferrero-Washington Theorem for the class group of the $\mathfrak{p}$-cyclotomic extension $\mathcal{F}$. For a more precise statement, let W be the Witt ring of $A/\mathfrak{p}$ and $\Lambda := W[[\Gamma]]$. Also, for any finite extension F_n/F, let $C\updownarrow_0(Fn)\{p\}$ denote the p-part of the group of classes of degree zero divisors of F_n and put $C(\mathcal{F})(\chi) := \varprojlim C\updownarrow_0(F_n)\{p\}$. Finally, let $FS_{/}F$ be the maximal abelian extension unramified outside $\{\infty, \mathfrak{p}\}$. Our main results are:

- there is a Stickelberger series $\Theta(X) \in \mathbb{Z}[[Gal(\mathcal{F}_S/F)]][X]]$ with the property that

it interpolates $L_S(s,\psi)$ (the complex L-function attached to $\psi : Gal(\mathcal{F}_S/F) \to \mathbb{C}^*$), the Carlitz-Goss zeta function ζ_A and the $\mathfrak{p}$-adic zeta function $L_{\mathfrak{p}}(X, y, \omega_{\mathfrak{p}}^i)$;

- for any non-trivial character χ of Δ, the χ-part of $C(\mathcal{F})$ is a finitely generated torsion Λ-module and one has

$$\mathrm{Fitt}_{\Lambda}(C(\mathcal{F})(\chi)) = (\Theta_{\chi}(1)),$$

where $\Theta_{\chi} \in \Lambda[[X]]$ is, up to a factor, the χ-component of the projection of Θ into $W[[Gal(\mathcal{F}/F)]][[X]]$ (the precise formula for Θ depends on whether χ is odd or even);

- for any non-trivial character χ of Δ, one has

$$\Theta_{\chi}(1) \not\equiv 0 (\mathrm{mod}\ p).$$

Bharathwaj Palvannan
Title: Algebraic analog of p-adic factorization formula
Abstract:

1. **Factorization formula** : We wish to prove the algebraic analog (involving Selmer groups) of various factorization formula (involving p-adic L-functions). We state an example of p-adic factorization (without explaining the notations for the sake of brevity).

$$\textbf{[Dasgupta]} \qquad \underbrace{\mathbf{L}_p(\mathcal{F},\mathcal{F},k,k,s)}_{\substack{\text{Hida's Rankin-Selberg}\\ \text{3-variable p-adic L-function}}} = \underbrace{\mathcal{L}_p(\mathrm{Sym}^2(f_k),s)}_{\substack{\text{Coates-Schmidt Sym}^2\\ \text{p-adic L-function}}} \cdot \underbrace{L_p(\chi^{-1},k-s)}_{\substack{\text{Kubota-Leopoldt}\\ \text{p-adic L-function}}}$$

   In particular, we wish to show that the results on the algebraic and analytic side are consistent with the main conjecture formulated by **Ralph Greenberg**.

2. **Control Theorems** :
   The previous objective leads us naturally to study the behavior of Selmer groups under specialization. The key is to study the *non-primitive Selmer groups* (**Greenberg-Vatsal**). We are led to study certain homological properties.

   (a) The dual of the Selmer group has no non-trivial pseudo-null submodules.

   (b) The projective dimension of the dual of the Selmer group at certain height 2 primes is finite.

Nobuo Sato
Title: On the Construction of Certain Ray Class Invariant and a Refinement of Zagier's Conjecture
Abstract: Let F be an imaginary quadratic number field, H the Hilbert class field of F. Zagier constructed a certain $\mathbb{C}/\mathbb{Z}(m)$-valued class invariant whose real/imaginary part is the partial zeta value $\zeta_F(m,\sigma)$ for $\sigma \in Gal(H/F)$. He also constructed a $\mathbb{C}/\mathbb{Q}(m)$-valued function on the m-th Bloch group which he called "The enhanced polylogarithm", and conjectured that there is a natural lift of his polylogarithm conjecture (The enhanced conjecture). His construction of the lifted partial zeta values uses the theory

of modular forms, and it is not clear how to construct such class invariants for other number fields. We show that Zagier's lifted partial zeta value is essentially the first partial derivative of Shintani L-function for imaginary quadratic number fields, and construct such invariants for the case where the number field F has a single complex place and H/F is an abelian extension such that no real places split. We then formulate Zagier's enhanced conjecture for such cases.

Florian Sprung
Title:
Abstract: Given a modular form of weight k and a prime p coprime to the level, one can attach p-adic L-functions, which encode special values of the complex L-function twisted by p-power Dirichlet characters. In view of BSD, these special values contain information about various arithmetic objects, such as Bloch-Kato-Safarevic-Tate groups. In the case in which the p^{th} Fourier coefficient a_p is a p-adic unit, the p-adic L-function is an Iwasawa function, i.e. converges on the closed unit disk, or equivalently, has bounded coefficient as a power series. When a_p is not a p-adic unit, there are two p-adic L-functions constructed in the work of Amice and Vélu, and independently Visik in the 1970's, but these p-adic L-functions are not Iwasawa.

In the early 2000's, Pollack discovered a decomposition of these p-adic L-functions into Iwasawa functions when $a_p = 0$. The present poster now gives a decomposition in the general supersingular case. It is more complex than the corresponding work of Pollack, and in fact not a generalization. In contrast to Pollack's pair of Iwasawa function, we obtain $k-1$ pairs. They are responsible for the behavior of the complex L-function at any of the $k-1$ special points at which we twist by p-power characters. An important open question is what the relation between these $k-1$ pairs and Pollack's pair is in the case $a_p = 0$. Can you find it?

Kwok-Wing Tsoi
Title: Congruences for CM forms
Abstract: In a recent work of M.Kakde, it was proved that a family of Λ-adic Hilbert modular Eisenstein series constructed Deligne-Ribet satisfies a congruence relation. Using this congruence, a q-expansion whose constant term is a non-commutative p-adic L-function was constructed. This poster aims to present that a similar congruence holds for a family of Λ-adic CM forms.

Fix ϕ to be a Hecke character over a quadratic imaginary field K. The CM form attached to ϕ has a q-expansion $\theta(q) = \sum_{\mathfrak{a}\subseteq O_K} \phi(\mathfrak{a})q^{N_{K/Q}(\mathfrak{a})}$. Using Hida Theory, one can write down the Λ-adic form that θ belongs to. The good news is that we can write down its q-expansion explicitly. By doing the base change, we can obtain a family of Λ-adic CM forms ranging over a tower of fields over $\mathbb{Q}$. We are going to examine the congruence between the q-expansion of this family.