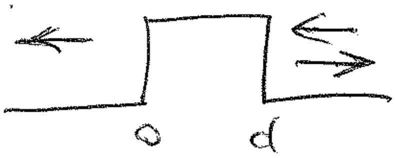

$$\begin{cases} \psi_R = e^{-ikx} + Re^{ikx} \\ \psi_C = Ce^{-iqx} + \Delta e^{iqx} \\ \psi_L = Te^{-ikx} \end{cases}$$

$$T = C + \Delta \quad \rightarrow \quad C + \Delta - T = 0$$

$$+kT = +q(C - \Delta) \quad \rightarrow \quad qC - q\Delta - kT = 0$$

$$e^{-ikd} + Re^{ikd} = Ce^{-iqd} + De^{iqd}$$

$$k(e^{-ikd} - Re^{ikd}) = q(Ce^{-iqd} - De^{iqd})$$

$$\begin{pmatrix} 1 & 1 & -1 & 0 \\ q & -q & -k & 0 \\ e^{-iqd} & e^{iqd} & 0 & -e^{ikd} \\ qe^{-iqd} & -qe^{iqd} & 0 & ke^{ikd} \end{pmatrix} \begin{pmatrix} C \\ \Delta \\ T \\ R \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ e^{-ikd} \\ ke^{-ikd} \end{pmatrix}$$

$$|A| = e^{ikd} \begin{vmatrix} 1 & 1 & -1 \\ q & -q & -k \\ qe^{-iqd} & -qe^{iqd} & 0 \end{vmatrix} + ke^{ikd} \begin{vmatrix} 1 & 1 & -1 \\ q & -q & -k \\ e^{-iqd} & e^{iqd} & 0 \end{vmatrix}$$

$$= e^{ikd}\left[qe^{-iqd}(-k-q) + qe^{iqd}(-k+q)\right] + ke^{ikd}\left[e^{-iqd}(-k-q) - e^{iqd}(-k+q)\right]$$

$$= e^{ikd} q\left[-k(e^{-iqd} + e^{iqd}) + q(-e^{-iqd} + e^{iqd})\right]$$

$$+ ke^{ikd}\left[+k(-e^{-iqd} + e^{iqd}) - q(e^{-iqd} + e^{iqd})\right]$$

$$= e^{ikd} q\left[-k2c + q2is\right] + e^{ikd} k\left[k \cdot 2is - q2c\right]$$

$$= 2e^{ikd}\left\{-qkc + iq^2 s + ik^2 s - kqc\right\}$$

$$= 2e^{ikd}\left[-2kqc + i(q^2 + k^2)s\right]$$

$$A_T = \begin{pmatrix} 1 & 1 & 0 & 0 \\ q & -q & 0 & 0 \\ e^{-iqd} & e^{iqd} & e^{-ikd} & -e^{ikd} \\ qe^{-iqd} & e^{iqd} & ke^{-ikd} & ke^{ikd} \end{pmatrix}$$

$$|A_T| = \left| \cancel{\begin{matrix} q & -q & 0 \\ e^{-iqd} & e^{iqd} & \end{matrix}} \right| \begin{vmatrix} -q & 0 & 0 \\ e^{iqd} & e^{-ikd} & -e^{ikd} \\ e^{iqd} & ke^{-ikd} & ke^{ikd} \end{vmatrix} -$$

$$- \begin{vmatrix} q & 0 & 0 \\ e^{-iqd} & e^{-ikd} & -e^{ikd} \\ qe^{-iqd} & ke^{-ikd} & ke^{ikd} \end{vmatrix} = -q\left[k+k\right] - q\left[k+k\right] = -4qk$$

$$T = \frac{-4qk}{2e^{ikd}\left[i(q^2+k^2)s - 2kqc\right]} = \frac{+2qke^{-ikd}}{-i(q^2+k^2)s + 2kqc}$$

$$\cancel{R} A_R = \begin{pmatrix} 1 & 1 & -1 & 0 \\ q & -q & -k & 0 \\ e^{-iqd} & e^{iqd} & 0 & e^{-ikd} \\ qe^{-iqd} & -qe^{iqd} & 0 & ke^{-ikd} \end{pmatrix}$$

$$|A_R| = -e^{-ikd} \begin{vmatrix} 1 & 1 & -1 \\ q & -q & -k \\ qe^{-iqd} & -qe^{iqd} & 0 \end{vmatrix} + ke^{-ikd} \begin{vmatrix} 1 & 1 & -1 \\ q & -q & -k \\ e^{-iqd} & e^{iqd} & 0 \end{vmatrix}$$

$$= -e^{-ikd}\left\{ qe^{-iqd}(-k-q) + qe^{iqd}(-k+q) \right\}$$

$$+ke^{-ikd}\left\{ e^{-iqd}(-k-q) - e^{iqd}(-k+q) \right\}$$

$$= -e^{-ikd} q\left[-k\left(e^{-iqd}+e^{iqd}\right)+q\left(e^{iqd}-e^{-iqd}\right)\right]$$

$$+ e^{-ikd} k\left[k\left(e^{iqd}-e^{-iqd}\right)-q\left(e^{iqd}+e^{-iqd}\right)\right]$$

$$= e^{-ikd} q\left[k2c - q2is\right] + e^{-ikd} k\left[k2is - q2c\right]$$

$$= 2e^{-ikd}\left[\cancel{kqc} - iq^2 s + ik^2 s - \cancel{qkc}\right]$$

$$= 2e^{-ikd}\left[i(k^2-q^2)s\right] = 2e^{-ikd}\, i(k^2-q^2)s$$

$$R = \frac{\cancel{2}e^{-ikd}\, i(k^2-q^2)s}{\cancel{2}e^{ikd}\left[i(q^2+k^2)s - 2kqc\right]} = \frac{i(k^2-q^2)s\, e^{-2ikd}}{-2kqc + i(q^2+k^2)s}$$

$$|R|^2 = \frac{(k^2-q^2)^2 s^2}{4k^2q^2c^2 + (q^2+k^2)^2 s^2}$$

$$\boxed{|R^{\leftarrow}|^2 = \frac{(k^2-q^2)^2}{(q^2+k^2)^2 + 4k^2q^2\cot^2(qd)}}$$

$\Rightarrow$ this is the same as for $|R^{\rightarrow}|^2$

$$|T|^2 = \frac{4q^2k^2}{4k^2q^2c^2 + (q^2+k^2)^2 s^2} = \frac{1}{\underbrace{c^2}_{1-s^2} + \left(\frac{q^2+k^2}{2kq}\right)^2 s^2}$$

$$D = 1 + s^2\left[\left(\frac{q^2+k^2}{2kq}\right)^2 - 1\right] = 1 + s^2\,\frac{q^4+k^4+2q^2k^2-4q^2k^2}{(2kq)^2}$$

$$= 1 + s^2\,\frac{q^4+k^4-2q^2k^2}{(2kq)^2} = 1 + s^2\,\frac{(q^2-k^2)^2}{(2kq)^2}$$

$$\boxed{|T^{\leftarrow}|^2 = \left[1 + \left(\frac{q^2-k^2}{2kq}\right)^2 \sin(qd)\right]^{-1}} \equiv |T^{\rightarrow}|^2$$