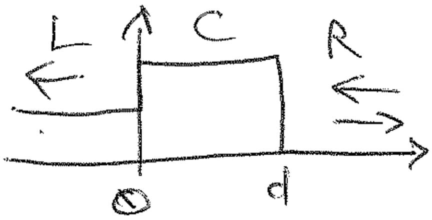
3.03.13 1

$R^{\leftarrow} = 1/B$

$A = i(-q^2 - ipk)is - iq(ik-p)c$
$= +s(q^2+ipk) + qc(k+ip) = (sq^2+qck) + i(spk+qcp)$
$= q(sq+ck) + ip(sk+qc)$

q

$B = -i(-q^2+ipk)is + iq(ik+p)c = s(-q^2+ipk) + q(-k+ip)c$
$= (-q^2 s - qkc) + i(spk+qpc) = -q(qs+kc) + ip(sk+qc)$



$$\begin{cases} \psi_R^{\leftarrow} = e^{-ipx} + R^{\leftarrow} e^{ipx} \\ \psi_C^{\leftarrow} = Ce^{-iqx} + De^{iqx} \\ \psi_L^{\leftarrow} = Te^{-ikx} \end{cases}$$

Flow to the left

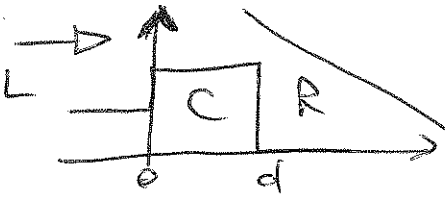
$\boxed{T = C + D}$ (1)

$-ikT = -iqC + iqD = -iq(C-D) \Rightarrow \boxed{kT = q(C-D)}$ (2)

$\boxed{Ce^{-iqd} + De^{iqd} = e^{-ipd} + Re^{ipd}}$ (3)

$-iq(Ce^{-iqd} - De^{iqd}) = -ip(e^{-ipd} - Re^{ipd})$ (+)

$\boxed{q(Ce^{-iqd} - De^{iqd}) = p(e^{-ipd} - Re^{ipd})}$ (4)



$\psi_L^{\rightarrow} = e^{ikx} + Re^{-ikx}$

$\psi_C^{\rightarrow} = Ce^{iqx} + De^{-iqx}$

$\psi_R^{\rightarrow} = Te^{ipx}$

$C_1 = Ce^{+iqd}$, $D_1 = De^{-iqd}$

$T_1 = Te^{ipd}$

$C_1 + D_1 = T_1$

$$\begin{cases} 1 + R = C + D \\ k(1-R) = q(C-D) \\ Ce^{iqd} + De^{-iqd} = Te^{ipd} \\ q(Ce^{iqd} - De^{-iqd}) = pTe^{ipd} \end{cases}$$

$$\begin{cases} 1 + R = C_1 e^{-iqd} + D_1 e^{iqd} \\ k(1-R) = q(C_1 e^{-iqd} - D_1 e^{iqd}) \\ \boxed{C_1 + D_1 = T_1} \ ; \ \{q(C_1 - D_1) = pT_1\} \ (p \leftrightarrow q) \end{cases}$$

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$$\begin{cases} C + D - T = 0 \\ qC - qD - kT = 0 \\ e^{-iqd}C + e^{iqd}D + 0 - e^{ipd}R = +e^{-ipd} \quad \oplus \\ qe^{-iqd}C - qe^{iqd}D + 0 + pe^{ipd}R = pe^{-ipd} \end{cases}$$

$$A = \begin{pmatrix} 1 & 1 & -1 & 0 \\ q & -q & -k & 0 \\ e^{-iqd} & e^{+iqd} & 0 & -e^{+ipd} \\ qe^{-iqd} & -qe^{iqd} & 0 & pe^{ipd} \end{pmatrix} \begin{pmatrix} C \\ D \\ T \\ R \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ e^{-ipd} \\ pe^{-ipd} \end{pmatrix} \quad \oplus$$

$$|A| = e^{+ipd} \begin{vmatrix} 1 & 1 & -1 \\ q & -q & -k \\ qe^{-iqd} & -qe^{iqd} & 0 \end{vmatrix} + pe^{ipd} \begin{vmatrix} 1 & 1 & -1 \\ q & -q & -k \\ e^{-iqd} & e^{iqd} & 0 \end{vmatrix}$$

along the 4th column

$$= e^{+ipd}\left\{qe^{-iqd}(-k-q) + qe^{iqd}(-k+q)\right\}^{\oplus}$$

$$+ pe^{ipd}\left\{e^{-iqd}(-k-q) - e^{iqd}(-k+q)\right\}^{\oplus} = A_1 + A_2 \,;$$

$$A_1 = e^{+ipd}q\left\{-k(e^{-iqd} + e^{iqd}) + q(-e^{-iqd} + e^{iqd})\right\}$$

$$= 2qe^{+ipd}\left\{-kc + iqs\right\}^{\oplus} = 2qe^{+ipd}(-kc + iqs)^{\oplus};$$

$$A_2 = pe^{ipd}\left\{+k(-e^{-iqd} + e^{iqd}) - q(e^{-iqd} + e^{iqd})\right\}^{\oplus}$$

$$= pe^{ipd}\left\{+2ksi - 2qc\right\}^{\oplus} = 2pe^{ipd}(-qc + iks)^{\oplus}$$

$$|A| = 2e^{ipd}\left\{q(-kc + iqs) + p(-qc + iks)\right\}^{\oplus}$$

$$= 2e^{ipd}\left\{-kqc - qpc + iq^2s + ipks\right\} = 2e^{ipd}\left[-(k+p)qc + i(q^2+pk)s\right] \quad \oplus$$

$$A_T = \begin{pmatrix} 1 & 1 & 0 & 0 \\ q & -q & 0 & 0 \\ e^{-iqd} & e^{iqd} & e^{-ipd} & -e^{ipd} \\ q e^{-iqd} & -q e^{iqd} & p e^{-ipd} & p e^{ipd} \end{pmatrix}$$

along the 1st row

$$|A_T| = \begin{vmatrix} -q & 0 & 0 \\ e^{iqd} & e^{-ipd} & -e^{ipd} \\ -q e^{iqd} & p e^{-ipd} & p e^{ipd} \end{vmatrix} - \begin{vmatrix} q & 0 & 0 \\ e^{-iqd} & e^{-ipd} & -e^{ipd} \\ q e^{-iqd} & p e^{-ipd} & p e^{ipd} \end{vmatrix}$$

$$= -q(p+p) - q(p+p) = -2q\,2p = -4qp$$

$$T = \frac{|A_T|}{|A|} = \frac{-4qp}{-2e^{ipd}\left[(k+p)qc - i(q^2+pk)s\right]}$$

$$\boxed{T = \frac{2qp\,e^{-ipd}}{(k+p)qc - i(q^2+pk)s}}$$ $(*)$

$$A_R = \begin{pmatrix} 1 & 1 & -1 & 0 \\ q & -q & -k & 0 \\ e^{-iqd} & e^{iqd} & 0 & e^{-ipd} \\ q e^{-iqd} & -q e^{iqd} & 0 & p e^{-ipd} \end{pmatrix}$$

along 3rd column

$$|A_R| = -1 \begin{vmatrix} q & -q & 0 \\ e^{-iqd} & e^{iqd} & e^{-ipd} \\ q e^{-iqd} & -q e^{iqd} & p e^{-ipd} \end{vmatrix} + k \begin{vmatrix} 1 & 1 & 0 \\ e^{-iqd} & e^{iqd} & e^{-ipd} \\ q e^{-iqd} & -q e^{iqd} & p e^{-ipd} \end{vmatrix}$$

$$= -\left\{ q\left(e^{iqd} p e^{-ipd} + q e^{-ipd} e^{iqd}\right) + q\left(p e^{-iqd} e^{-ipd} - q e^{-iqd} e^{-ipd}\right) \right\} +$$

$$+k\left\{\left(pe^{iqd}e^{-ipd}+qe^{iqd}e^{-ipd}\right)-\left(pe^{-iqd}e^{-ipd}-qe^{-iqd}e^{-ipd}\right)\right\}_{\oplus}$$

$$=-q\left\{(p+q)e^{iqd}e^{-ipd}+(p-q)e^{-iqd}e^{-ipd}\right\}_{\oplus}$$

$$+k\left\{(p+q)e^{iqd}e^{-ipd}-(p-q)e^{-iqd}e^{-ipd}\right\}_{\oplus}$$

$$=-qe^{-ipd}\left[(p+q)e^{iqd}+(p-q)e^{-iqd}\right]_{\oplus}$$

$$+ke^{-ipd}\left[(p+q)e^{iqd}-(p-q)e^{-iqd}\right]_{\oplus}$$

$$=-qe^{-ipd}\left[2pc+2iqs\right]_{\oplus}+ke^{-ipd}\left[2ips+q2c\right]_{\oplus}$$

$$=2e^{-ipd}\left\{-q(pc+iqs)+k(qc+ips)\right\}_{\oplus}$$

$$=2e^{-ipd}\left[-qpc-iq^2s+kqc+ikps\right]_{\oplus}=2e^{-ipd}\left[-q(p-k)c-i(q^2-kp)s\right]$$

$$=-2e^{-ipd}\left[q(p-k)c+i(q^2-kp)s\right]$$

$$R^{\leftarrow}=\frac{|A_R|}{|A|}=\frac{-2e^{-ipd}\left[q(p-k)c+i(q^2-kp)s\right]}{2e^{ipd}\left[-q(p+k)c+i(q^2+kp)s\right]}$$

$$\boxed{R^{\leftarrow}=e^{-2ipd}\,\frac{q(p-k)c+i(q^2-kp)s}{q(p+k)c-i(q^2+kp)s}}\quad (*)$$

If $U=0\rightarrow\boxed{p=k}$

$$R^{\leftarrow}=e^{-2ikd}\,\frac{+i(-q^2+k^2)s}{-2qkc+i(q^2+k^2)s}$$