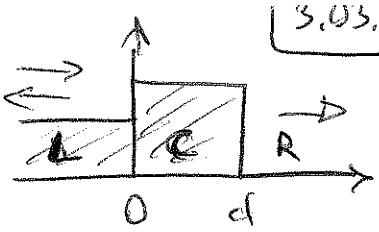
3.03.13



$$\begin{cases} \psi_L^{\to} = e^{ikx} + R e^{-ikx} \\ \psi_C^{\to} = C e^{iqx} + D e^{-iqx} \\ \psi_R^{\to} = T e^{ipx} \end{cases}$$

Flow to the right

1

$$\boxed{\begin{aligned} &1+R = C+D \\ &k(1-R) = q(C-D) \\ &Ce^{iqd} + De^{-iqd} = Te^{ipd} \\ &q(Ce^{iqd} - De^{-iqd}) = pTe^{ipd} \end{aligned}}$$

Matrix form:

$$\begin{pmatrix} 1 & 1 & -1 & 0 \\ q & -q & k & 0 \\ e^{iqd} & e^{-iqd} & 0 & -e^{ipd} \\ qe^{iqd} & -qe^{-iqd} & 0 & -pe^{ipd} \end{pmatrix} \begin{pmatrix} C \\ D \\ R \\ T \end{pmatrix} = \begin{pmatrix} 1 \\ k \\ 0 \\ 0 \end{pmatrix}$$

$$|A| = \left(\text{along the 4th column}\right) = +e^{ipd} \begin{vmatrix} 1 & 1 & -1 \\ q & -q & k \\ qe^{iqd} & -qe^{-iqd} & 0 \end{vmatrix} - pe^{ipd} \begin{vmatrix} 1 & 1 & -1 \\ q & -q & k \\ e^{iqd} & e^{-iqd} & 0 \end{vmatrix}$$

$$= e^{ipd}\left[kqe^{-iqd} + kqe^{iqd} - (-q^2e^{-iqd} + q^2e^{iqd})\right]^{\oplus} -$$

$$- pe^{ipd}\left[-ke^{-iqd} + ke^{+iqd} - (qe^{-iqd} + qe^{iqd})\right]^{\oplus}$$

$$= e^{ipd}\left[2kqc - q^2 2is\right]^{\oplus} - pe^{ipd}\left[2kis - 2qc\right]^{\oplus}$$

$$= 2e^{ipd}\left[kqc - iq^2s - ipks + pqc\right] = 2e^{ipd}\left[q(p+k)c - i(q^2+pk)s\right]^{\oplus}$$

$$A_T = \begin{pmatrix} 1 & 1 & -1 & 1 \\ q & -q & k & k \\ e^{iqd} & e^{-iqd} & 0 & 0 \\ qe^{iqd} & -qe^{-iqd} & 0 & 0 \end{pmatrix}$$

By the 4th column

$$|A_T| = -\begin{vmatrix} q & -q & k \\ e^{iqd} & e^{-iqd} & 0 \\ qe^{iqd} & -qe^{-iqd} & 0 \end{vmatrix} + k \begin{vmatrix} 1 & 1 & -1 \\ e^{iqd} & e^{-iqd} & 0 \\ qe^{iqd} & -qe^{-iqd} & 0 \end{vmatrix}$$

$$= -k\left[-q-q\right] + k\left[(-1)(-q-q)\right] = 2qk + 2qk = 4qk$$

$$T^{\rightarrow} = \frac{4qk}{2e^{ipd}\left[q(p+k)c - i(q^2+pk)s\right]}$$

$$\boxed{T^{\rightarrow} = \frac{2qk\, e^{-ipd}}{q(p+k)c - i(q^2+pk)s}}$$

$$A_R = \begin{pmatrix} 1 & 1 & 1 & 0 \\ q & -q & k & 0 \\ e^{iqd} & e^{-iqd} & 0 & -e^{ipd} \\ qe^{iqd} & -qe^{-iqd} & 0 & -pe^{ipd} \end{pmatrix}$$

along the 4th column

$$|A_R| = +e^{ipd} \begin{vmatrix} 1 & 1 & 1 \\ q & -q & k \\ qe^{iqd} & -qe^{-iqd} & 0 \end{vmatrix} - pe^{ipd} \begin{vmatrix} 1 & 1 & 1 \\ q & -q & k \\ e^{iqd} & e^{-iqd} & 0 \end{vmatrix}$$

$$= e^{ipd}\left\{qe^{iqd}(k+q) + qe^{-iqd}(k-q)\right\} - pe^{ipd}\left\{e^{iqd}(k+q) - e^{-iqd}(k-q)\right\}$$

$$= e^{ipd}\cancel{e}\left\{qk\left(e^{iqd}+e^{-iqd}\right)+q^2\left(e^{iqd}-e^{-iqd}\right)\right\}$$

$$-p\,e^{ipd}\left\{k\left(e^{iqd}-e^{-iqd}\right)+q\left(e^{iqd}+e^{-iqd}\right)\right\}$$

$$= e^{ipd}\left\{\underline{2qkc}+2q^2 is-pk2is-\underline{pq2c}\right\}$$

$$= 2e^{ipd}\left[q(k-p)c+i(q^2-pk)s\right]$$

$$R^{\rightarrow} = \frac{\cancel{2e^{ipd}}\left[q(k-p)c+i(q^2-pk)s\right]}{\cancel{2e^{ipd}}\left[q(k+p)c-i(q^2+pk)s\right]} \qquad \boxed{R^{\rightarrow} = \frac{q(k-p)c+i(q^2-pk)s}{q(k+p)c-i(q^2+pk)s}}$$

$$\boxed{u=0} \longrightarrow \boxed{p=k}$$

$$\boxed{R^{\rightarrow} = \frac{i(q^2-k^2)s}{2qkc-i(q^2+k^2)s} \qquad T^{\rightarrow} = \frac{2qk\,e^{-ikd}}{2qkc-i(q^2+k^2)s}}$$

<u>correct!</u>