asymmetric barrier

CURRENT

7.05.15

$$R = i(ipk - q^2)\,is - iq(p+iq)c = -(ipk - q^2)s - q(ip-q)c$$

$$\cancel{1 = (q^2 s + q^2}$$

$$|T^{\leftarrow}|^2 = \left(\frac{p}{k}\right)^2 |T^{\rightarrow}|^2 \qquad \boxed{|T^{\leftarrow}|^2 = \frac{p^2}{k^2}|T^{\rightarrow}|^2}$$

$$j_L^{\rightarrow} = \frac{e\hbar k}{m}\left(1 - |R^{\rightarrow}|^2\right), \quad j_R^{\rightarrow} = \frac{e\hbar p}{m}|T^{\rightarrow}|^2$$

$$j_L^{\leftarrow} = -\frac{e\hbar k}{m}|T^{\leftarrow}|^2 = -\frac{e\hbar k}{m}\frac{p^2}{k^2}|T^{\rightarrow}|^2 = -\frac{e\hbar p^2}{mk}|T^{\rightarrow}|^2$$

$$\boxed{j_L^{tot} = \frac{e\hbar k}{m}\left[1 - |R^{\rightarrow}|^2 - |T^{\leftarrow}|^2\right]}$$

$$j_R^{\leftarrow} = -\frac{e\hbar p}{m}\left(1 - |R^{\leftarrow}|^2\right)$$

$$\boxed{j_R^{tot} = \frac{e\hbar p}{m}\left[|T^{\rightarrow}|^2 - 1 + |R^{\leftarrow}|^2\right]}$$

$$j_L^{tot} = \frac{e\hbar k}{m}\left[1 - \frac{q^2(qs-kc)^2 + p^2(qc+ks)^2}{q^2(qs+kc)^2 + p^2(qc+ks)^2} - \frac{4q^2p^2}{\ldots}\right]$$

$$= \frac{e\hbar k}{m}\,\frac{q^2(qs+kc)^2 + \cancel{p^2(qc+ks)^2} - q^2(qs-kc)^2 - \cancel{p^2(qc+ks)^2} - 4q^2p^2}{\ldots\ldots} = \frac{e\hbar k}{m}\frac{A}{B}$$

$$A = q^2(qs + \cancel{ks} + qs - \cancel{kc})(\cancel{qs} + kc - \cancel{qs} + kc) - 4q^2p^2$$

$$= q^2(2qs)(2kc) - 4q^2p^2 = 4q^3kcs - 4q^2p^2 = 4q^2(qkcs - p^2)$$

$$\boxed{j_L^{tot} = \frac{e\hbar k}{m}\,\frac{4q^2(qkcs - p^2)}{q^2(qs+kc)^2 + p^2(qc+ks)^2}}$$

$$j_R^{tot} = \frac{e\hbar p}{m}\left[\cancel{-1} + \frac{4k^2q^2}{\ldots} \cancel{+1}\right] = \frac{e\hbar p}{m}\,\frac{4k^2q^2}{\ldots}$$

$\underline{U < E < U + V_0}$ $\underline{q \to iq \text{ only}}$ $\boxed{s \to is,\ c \to c}$

$$T^{\rightarrow} = \frac{2iqk e^{-ipd}}{iq(p+k)c - i(-q^2+pk)is}$$

$$= \frac{2iqk e^{-ipd}}{iq(p+k)c + (-q^2+pk)s}, \quad \boxed{|T^{\rightarrow}|^2 = \frac{4q^2k^2}{(q^2-pk)^2 s^2 + q^2(p+k)^2c^2}}$$

$$R^{\rightarrow} = \frac{iq(k-p)c + i(-q^2-pk)is}{iq(k+p)c - i(-q^2+pk)is} = \frac{iq(k-p)c + (q^2+pk)s}{iq(k+p)c + (-q^2+pk)s}$$

$$\boxed{|R^{\rightarrow}|^2 = \frac{q^2(k-p)^2c^2 + (q^2+pk)^2 s^2}{q^2(k+p)^2c^2 + (q^2-pk)^2 s^2}}$$

$$T^{\leftarrow} = \frac{2iqp e^{-ipd}}{iq(k+p)c - i(-q^2+pk)is} = \frac{2iqp e^{-ipd}}{iq(k+p)c + (-q^2+pk)s}$$

$$\boxed{|T^{\leftarrow}|^2 = \frac{4q^2p^2}{q^2(k+p)^2c^2 + (q^2-pk)^2 s^2}} \boxed{\equiv \frac{p^2}{k^2}|T^{\rightarrow}|^2}$$

$$R^{\leftarrow} = e^{-2ipd}\frac{iq(p-k)c + i(-q^2-kp)is}{iq(p+k)c - i(-q^2+kp)is} = e^{-2ipd}\frac{iq(p-k)c + (q^2+pk)s}{iq(p+k)c + (-q^2+kp)s}$$

$$\boxed{|R^{\leftarrow}|^2 = \frac{q^2(p-k)^2c^2 + (q^2+pk)^2 s^2}{q^2(p+k)^2c^2 + (q^2-pk)^2 s^2}} \boxed{\equiv |R^{\rightarrow}|^2}$$

$$1-|R^{\rightarrow}|^2-|T^{\leftarrow}|^2 = 1-\frac{q^2(k-p)^2c^2+(q^2+pk)^2s^2}{q^2(k+p)^2c^2+(q^2-pk)^2s^2}-\frac{4q^2p^2}{\ldots}$$

$$= \frac{1}{\ldots}\left\{\underbrace{q^2(k+p)^2c^2}+(q^2-pk)^2s^2-\underbrace{q^2(k-p)^2c^2}-(q^2+pk)^2s^2-\right.$$
$$\left.-4q^2p^2\right\}$$

$$\{\ldots\} = q^2c^2\left[(k+p)^2-(k-p)^2\right]+s^2\left[(q^2-pk)^2-(q^2+pk)^2\right]-4q^2p^2$$

$$= q^2c^2\cdot 4kp - s^2\cdot 4q^2pk - 4q^2p^2 = 4q^2kp\underbrace{(c^2-s^2)}_{1}-4q^2p^2$$

$$= 4q^2kp-4q^2p^2 = 4q^2p(k-p)$$

$$\boxed{j_L^{tot} = \frac{e\hbar k}{m}\frac{4q^2p(k-p)}{\ldots}}$$

$$|T^{\rightarrow}|^2-1+|R^{\leftarrow}|^2 = \frac{4q^2k^2}{\ldots}-1+\frac{q^2(p-k)^2c^2+(q^2+pk)^2s^2}{\ldots}$$

$$= \frac{1}{\ldots}\left\{4q^2k^2-\underbrace{q^2(k+p)^2c^2}-\underbrace{(q^2-pk)^2s^2}+\underbrace{q^2(p-k)^2c^2}+\underbrace{(q^2+pk)^2s^2}\right\}$$

$$= \frac{1}{\ldots}\left\{4q^2k^2 - q^2c^2\underbrace{\left[(k+p)^2-(p-k)^2\right]}_{4kp} - s^2\underbrace{\left[(q^2-pk)^2-(q^2+pk)^2\right]}_{-4q^2pk}\right\}$$

$$= \frac{1}{\ldots}\left\{4q^2k^2-4q^2kpc^2+4q^2kps^2\right\} = \frac{4q^2k}{\ldots}\left\{k-pc^2+ps^2\right\}$$

$$= \frac{4q^2k}{\ldots}(k-p) \rightarrow \boxed{j_R^{tot} = \frac{e\hbar p}{m}\frac{4q^2k(k-p)}{\ldots}}\boxed{=j_L^{tot}}$$