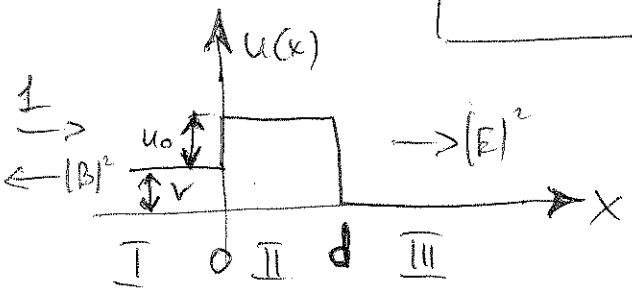
$\boxed{\text{Asymmetric potential barrier}}$ (I) Right



(I) $\psi_I = e^{ikx} + B e^{-ikx}$, $k = \frac{1}{\hbar}\sqrt{2m(\mathcal{E}-V)}$

(II) $\psi_{II} = C e^{iqx} + D e^{-iqx}$, $q = \frac{1}{\hbar}\sqrt{2m(\mathcal{E}-u_0-V)}$

(III) $\psi_{III} = E\, e^{ipx}$, $p = \frac{1}{\hbar}\sqrt{2m\mathcal{E}}$

Continuity at $x=0$:

$$\begin{cases} 1+B = C+D \\ ik(1-B) = iq(C-D) \end{cases} \quad (1)$$

Continuity at $x=d$

$$\begin{cases} Ce^{iqd}+De^{-iqd} = Ee^{ipd} \\ iq(Ce^{iqd}-De^{-iqd}) = ipEe^{ipd} \end{cases} \quad (2)$$

From (2): $\alpha = Ce^{iqd}$, $\beta = De^{-iqd}$

$$\alpha+\beta = \frac{q}{p}(\alpha-\beta), \quad \frac{\alpha}{\beta}+1 = \frac{q}{p}\left(\frac{\alpha}{\beta}-1\right), \quad \frac{\alpha}{\beta}\left(1-\frac{q}{p}\right) = -1-\frac{q}{p}$$

$$\frac{\alpha}{\beta} = -\frac{1+q/p}{1-q/p} = -\frac{p+q}{p-q} = \frac{q+p}{q-p}, \quad \alpha = \frac{q+p}{q-p}\beta$$

$$\hookrightarrow Ce^{iqd} = \frac{q+p}{q-p}De^{-iqd}, \quad \boxed{C = \frac{q+p}{q-p}e^{-2iqd}D} = \varkappa D \quad (3)$$

From (1):

$$B = C+D-1 = -\frac{q}{k}(C-D)+1$$

$$C\left[1+\frac{q}{k}\right]+D\left[1-\frac{q}{k}\right] = 2, \quad C(k+q)+D(k-q) = 2k$$

$$\hookrightarrow D(k-q) + \frac{(q+p)(q+k)}{q-p}e^{-2iqd}D = 2k$$

$$D = \frac{2k}{(k-q)+\frac{(q+p)(q+k)}{q-p}e^{-2iqd}} = \frac{2k(q-p)}{(k-q)(q-p)+(q+p)(q+k)e^{-2iqd}}$$

This is one possible expression for the D.

Then,

$$E e^{ipd} = C e^{iqd} + D e^{-iqd}$$

$$= \frac{q+p}{q-p} e^{-iqd} D + D e^{-iqd} = \left(\frac{q+p}{q-p} + 1\right) D e^{-iqd} = \frac{2q}{q-p} e^{-iqd} D,$$

$$e^{ipd} E = \frac{2q}{q-p} e^{-iqd} \frac{2k(q-p)}{\cdots\cdots} = \frac{4kq\, e^{-iqd}}{\cdots},$$

$$\boxed{E = \frac{4kq\, e^{-iqd} e^{-ipd}}{-(q-k)(q-p) + (q+k)(q+p) e^{-2iqd}}}$$

The denominator here,

$$\text{Den.} = -(q^2+kp) + (kq+qp) + (q^2+kp) e^{-\cdots} + (qp+kq) e^{-\cdots}$$

$$= (q^2+kp)(e^{-\cdots} - 1) + q(k+p)(1 + e^{-\cdots})$$

$$= e^{-iqd}\left[(q^2+kp)(\underbrace{e^{-iqd} - e^{iqd}}_{-2i\sin qd}) + q(k+p)(\underbrace{e^{iqd} + e^{-iqd}}_{2\cos qd})\right]$$

$$= 2e^{-iqd}\left[-i(q^2+kp)\sin qd + q(k+p)\cos qd\right]$$

$$\hookrightarrow \boxed{E = \frac{2kq\, e^{-ipd}}{-i(q^2+kp)\sin qd + q(p+k)\cos qd}}$$

To find B,

$$B = C + D - 1 = \left(\frac{q+p}{q-p} e^{-2iqd} + 1\right) \cdot \frac{2k(q-p)}{-(q-k)(q-p) + (q+k)(q+p) e^{-2iqd}}$$

$$-1 = \frac{2k\left[(q+p) e^{-2iqd} + (q-p)\right]}{-(q-k)(q-p) + (q+k)(q+p) e^{-2iqd}} - 1$$

$$= \cancel{\frac{2k}{\text{Den}}\left[(q+p) e^{-2iqd} + (q-p) - (q-k)(q-p) + (q+k)(q+p) e^{-2iqd}\right]}$$

$$= \frac{1}{\cdots}\left\{2k(q+p) e^{\cdots} + 2k(q-p) + (q-k)(q-p) - (q+k)(q+p) e^{\cdots}\right\}$$

$$= \frac{1}{\cdots}\left\{(q+p)e^{\cdots}(2k-q-k)+(q-p)(2k+q-k)\right\}$$

$$= \frac{1}{\cdots}\left\{(q+p)(k-q)e^{\cdots}+(q-p)(k+q)\right\}$$

$$= \frac{(q+p)(k-q)e^{-2iqd}+(q-p)(k+q)}{-(q-k)(q-p)+(q+k)(q+p)e^{-2iqd}} = \frac{B_1}{B_2}$$

where

$$B_1 = (-q^2+pk)e^{\cdots} + (qk-pq)e^{\cdots} + (q^2-pk) + (-pq+qk)$$

$$= (-q^2+pk)(e^{\cdots}-1) + q(k-p)(e^{\cdots}+1)$$

$$= e^{-iqd}\left[(pk-q^2)\underbrace{(e^{-iqd}-e^{iqd})}_{-2i\sin qd} + q(k-p)\underbrace{(e^{-iqd}+e^{iqd})}_{2\cos qd}\right]$$

$$= 2e^{-iqd}\left[i(q^2-pk)\sin qd + q(k-p)\cos qd\right],$$

$$B_2 = (-q^2-pk)\cancel{e} + (qk+pq)\cancel{e} + (q^2+pk)e + (+pq+qk)e$$

$$= (+q^2+pk)(e^{\cdots}-1) + (qk+pq)(e^{\cdots}+1)$$

$$= e^{-iqd}\left[(+q^2+pk)(e^{-iqd}-e^{iqd}) + q(k+p)(e^{-iqd}+e^{iqd})\right]$$

$$= 2e^{-iqd}\left[-i(q^2+pk)\sin qd + q(k+p)\cos qd\right]$$

$$\boxed{B = \frac{i(q^2-pk)\sin qd + q(k-p)\cos qd}{-i(q^2+pk)\sin qd + q(k+p)\cos qd}}$$

or

$$\cancel{B = \frac{-(q^2-pk) + iq(k-p)\,\mathrm{ctan}(qd)}{+(q^2+pk) + iq(k+p)\,\mathrm{ctan}(qd)}}$$

Now we should be able to calculate the T & R.

(i) $\underline{E > U_0 + V}$ k and q are both real. Then,

$$T = |E|^2 = \frac{4k^2q^2 e^{-ipd}e^{ipd}}{[q(p+k)c - i(q^2+kp)s][q(p+k)c + i(q^2+kp)s]} =$$

$$= \frac{4k^2q^2}{\left[(q(p+k)C)^2 + ((q^2+kp)S)^2\right]}$$

$$= \frac{4k^2q^2}{q^2(p+k)^2(1-s^2) + (q^2+kp)^2 s^2} = \frac{4k^2q^2}{q^2(p+k)^2 + \left[(q^2+kp)^2 - q^2(p+k)^2\right] s^2}$$

$$= \frac{4k^2q^2}{q^2(p+k)^2}\left[1 + \left(\frac{(q^2+kp)^2}{q^2(p+k)^2} - 1\right)\sin^2 qd\right]^{-1},$$ ✓

$$\boxed{T_{\rightarrow} = \frac{4k^2}{(p+k)^2}\left[1 + \left\{\frac{(q^2+kp)^2}{q^2(p+k)^2} - 1\right\}\sin^2 qd\right]^{-1}}$$ ✓

Also: $\{\ldots\} \equiv \frac{(q^2-p^2)(q^2-k^2)}{q^2(p+k)^2}$ ✓

If $p=k$ ($V=0$), then $T_{\rightarrow} = \left[1 + \left(\frac{(q^2+k^2)^2}{q^2 \cdot 4k^2} - 1\right)\sin^2 qd\right]^{-1}$

with $(\ldots) = \frac{1}{4q^2k^2}\left((q^2+k^2)^2 - 4k^2q^2\right) = \frac{1}{4q^2k^2}(q^2+k^2-2kq)(q^2+k^2+2kq)$

$= \frac{1}{4q^2k^2}(q-k)^2(q+k)^2 = \frac{(q^2-k^2)^2}{4q^2k^2}$, the same result as for the symmetric case.

The reflection (for the overall direction $\rightarrow$):

$$R_{\rightarrow} = |B|^2 = \frac{-(q^2-pk) + iq(k-p)CT}{(q^2+pk) + iq(k+p)CT} \times \frac{-(q^2-pk) - iq(k-p)CT}{(q^2+pk) - iq(k+p)CT}$$

$$= \frac{(q^2-pk)^2 + (q(k-p)CT)^2}{(q^2+pk)^2 + (q(k+p)CT)^2}$$

$$= \frac{(q^2-pk)^2 + q^2(k-p)^2\frac{\cos^2(qd)}{\sin^2(qd)}}{(q^2+pk)^2 + q^2(k+p)^2\frac{C^2}{S^2}} = \frac{(q^2-pk)^2 S^2 + q^2(k-p)^2(1-S^2)}{(q^2+pk)^2 S^2 + q^2(k+p)^2(1-S^2)}$$ ✓

$$= \frac{q^2(k-p)^2 + \left[(q^2-pk)^2 - q^2(k-p)^2\right]S^2}{q^2(k+p)^2 + \left[(q^2+pk)^2 - q^2(k+p)^2\right]S^2}$$ ✓

where $[\ldots]_{up} = q^4 - \cancel{2q^2pk} + p^2k^2 - q^2k^2 - q^2p^2 + \cancel{2q^2kp} = q^2(q^2-k^2) + p^2(k^2-q^2)$

$= (q^2-k^2)(q^2-p^2)$

and $[\,..\,]_{dw} = q^4 + \cancel{2q^2pk} + p^2k^2 - q^2k^2 - q^2p^2 - \cancel{2q^2pk} = q^2(q^2-k^2) + p^2(k^2-q^2)$

$= (q^2-k^2)(q^2-p^2)$, i.e. the same,

$$\boxed{R_{\rightarrow} = \frac{q^2(k-p)^2 + (q^2-k^2)(q^2-p^2)\, S^2}{q^2(k+p)^2 + (q^2-k^2)(q^2-p^2)\, S^2}}$$

Let us calculate their sum:

$$R_{\rightarrow} + T_{\rightarrow} = \frac{q^2(k-p)^2 + (q^2-k^2)(q^2-p^2)S^2}{q^2(k+p)^2 + (q^2-k^2)(q^2-p^2)S^2} + \frac{4k^2}{\cancel{(p+k)^2}}\,\frac{q^2\cancel{(p+k)^2}}{q^2(p+k)^2 + (q^2-p^2)(q^2-k^2)S^2}$$

$$= \frac{q^2(k-p)^2 + (q^2-k^2)(q^2-p^2)S^2 + 4k^2q^2}{q^2(k+p)^2 + (q^2-k^2)(q^2-p^2)\, S^2}$$

Here $q^2(k-p)^2 + 4k^2q^2 = q^2\left((k-p)^2 + 4k^2\right) = q^2\left(k^2 - 2kp + p^2 + 4k^2\right)$

$= q^2(5k^2 - 2kp + p^2)$

so, $\boxed{R_{\rightarrow} + T_{\rightarrow} \neq 1}$, but is $= \dfrac{q^2[(k-p)^2+4k^2] + (q^2-k^2)(q^2-p^2)S^2}{q^2(k+p)^2 + (q^2-k^2)(q^2-p^2)S^2}$

The total current in III is $\boxed{j_{III}^{\rightarrow} = \frac{\hbar p}{m}|E|^2 = \frac{\hbar p}{m} T_{\rightarrow}}$

The current in I:

$$\boxed{j_I^{\rightarrow} = \frac{\hbar k}{m}\left(|B|^2 - 1\right)}$$

Current for the other solution of the same energy ($\leftarrow$) will be done later.

(ii) $V < \mathcal{E} < U_0 + V$. In this case: $\boxed{\begin{array}{l} k - \text{real}, \quad p - \text{real} \\ q = i\lambda, \ \lambda = \frac{1}{\hbar}\sqrt{2m(V+U_0-\mathcal{E})} \end{array}}$

$$E = \frac{2ik\lambda\, e^{-ipd}}{-i(kp-\lambda^2)\sin(i\lambda d) + i\lambda(p+k)\cos(i\lambda d)} = \frac{2ik\lambda\, e^{-ipd}}{(kp-\lambda^2)\sinh(\lambda d) + i\lambda(p+k)\cosh(\lambda d)}$$

$$T_{\rightarrow} = |E|^2 = \frac{4k^2\lambda^2}{(kp-\lambda^2)^2S^2 + (\lambda(p+k)C)^2} = \frac{4k^2\lambda^2}{(kp-\lambda^2)^2S^2 + \lambda^2(p+k)^2(1+S^2)}$$

$$= \frac{4k^2\lambda^2}{\lambda^2(p+k)^2 + \left[(kp-\lambda^2)^2 + \lambda^2(p+k)^2\right]S^2}$$

where

$$[\ldots] = k^2p^2 - \cancel{2\lambda^2 kp} + \lambda^4 + \lambda^2 p^2 + \lambda^2 k^2 + \cancel{2\lambda^2 pk} = \lambda^2(\lambda^2+p^2) + k^2(\lambda^2+p^2)$$
$$= (\lambda^2+p^2)(\lambda^2+k^2)$$

$$\boxed{T_{\to} = \frac{4k^2\lambda^2}{\lambda^2(p+k)^2 + (\lambda^2+p^2)(\lambda^2+k^2)\sinh^2(\lambda d)}}$$

$$B = \frac{i(-\lambda^2-pk)\sin(i\lambda d) + i\lambda(k-p)\cos(i\lambda d)}{-i(-\lambda^2+pk)\sin(i\lambda d) + i\lambda(k+p)\cos(i\lambda d)} = \frac{(\lambda^2+pk)\sinh(\lambda d) + i\lambda(k-p)\cosh(\lambda d)}{(-\lambda^2+pk)\sinh(\lambda d) + i\lambda(k+p)\cosh(\lambda d)}$$

$$R_{\to} = |B|^2 = \frac{(\lambda^2+pk)^2 s^2 + \lambda^2(k-p)^2 c^2}{(-\lambda^2+pk)^2 s^2 + \lambda^2(k+p)^2 c^2} = \frac{(\lambda^2+pk)^2 s^2 + \lambda^2(k-p)^2(1+s^2)}{(-\lambda^2+pk)^2 s^2 + \lambda^2(k+p)^2(1+s^2)}$$

$$= \frac{[(\lambda^2+pk)^2 + \lambda^2(k-p)^2]\, s^2 + \lambda^2(k-p)^2}{[(-\lambda^2+pk)^2 + \lambda^2(k+p)^2]\, s^2 + \lambda^2(k+p)^2},$$

where $\mathcal{E}\ (\lambda^2+pk)^2 + \lambda^2(k-p)^2 = \lambda^4 + \cancel{2\lambda^2 pk} + p^2k^2 + \lambda^2k^2 + \lambda^2p^2 - \cancel{2\lambda^2 kp}$

$$= \lambda^2(\lambda^2+k^2) + p^2(k^2+\lambda^2) = (\lambda^2+k^2)(\lambda^2+p^2);$$

the other $[\ldots]$ is the same $(p \to -p)$:

$$\boxed{R_{\to} = \frac{\lambda^2(k-p)^2 + (\lambda^2+k^2)(\lambda^2+p^2)\sinh^2(\lambda d)}{\lambda^2(k+p)^2 + (\lambda^2+k^2)(\lambda^2+p^2)\sinh^2(\lambda d)}}$$

~~We~~ (iiii) $0 < \mathcal{E} < V$ We shall not consider this case assuming V is small $(V \to 0)$.

[In fact, see IV.]

Introducing a common momentum $\boxed{k}$ (a.u.)

I: $\mathcal{E} = \frac{1}{2}k^2 + eV$ $\boxed{0 \le k \le k_F}$, $\mathcal{E}_F = \frac{1}{2}k_F^2$

II $q = \sqrt{2(\mathcal{E} - U_0 - eV)} = \sqrt{2(\mathcal{E}-eV) - 2U_0} = \sqrt{k^2 - 2U_0}$

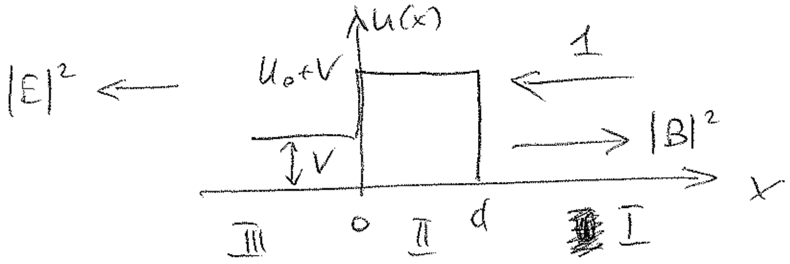
III $p = \sqrt{2\mathcal{E}} = \sqrt{k^2 + 2eV}$

And $\sum_{0<k<k_F} \ldots \to \int_0^{\mathcal{E}_F} \mathcal{D}(\mathcal{E}') \ldots d\mathcal{E}'$ with the usual DOS, θ

Or: $\sum_k \to \frac{2}{(2\pi)^3} \int_0^{k_F} \ldots dk$ for this normalisation of the wave functions.

II Left (from Right)

Now we consider the transport to the left.



$$\psi_{I} = e^{-ipx} + B e^{ipx}$$ ← this guarantees the same energy!

$$\psi_{II} = C e^{-iqx} + D e^{iqx}$$

$$\psi_{III} = E e^{-ikx}$$

Continuity at $x = 0$

$$\begin{cases} C + D = E & (1) \\ -iq(C - D) = -ikE \end{cases}$$

Continuity at $x = d$

$$\begin{cases} e^{-ipd} + B e^{ipd} = C e^{-iqd} + D e^{iqd} & (2) \\ -ip(e^{-ipd} - B e^{ipd}) = -iq(C e^{-iqd} - D e^{iqd}) \end{cases}$$

Rewrite Eqs. (2):

$1 + B_1 = C_1 + D_1$, where $\boxed{B_1 = B e^{2ipd}, \quad C_1 = C e^{-iqd} e^{ipd}, \quad D_1 = D e^{iqd} e^{ipd}}$ (3)

and

$$p(1 - B_1) = q(C_1 - D_1),$$

i.e. we have:

$$\boxed{\begin{cases} 1 + B_1 = C_1 + D_1 \\ p(1 - B_1) = q(C_1 - D_1) \end{cases}} \quad (4)$$

Eqs. (1) read then:

$$C_1 e^{iqd} e^{-ipd} + D_1 e^{-iqd} e^{-ipd} = E \to C_1 e^{iqd} + D_1 e^{-iqd} = \overbrace{E e^{ipd}}^{E_1 e^{ikd}}$$

and

$$q(C_1 e^{iqd} e^{-ipd} - D_1 e^{-iqd} e^{-ipd}) = kE \to q(C_1 e^{iqd} - D_1 e^{-iqd}) = k E_1 e^{ikd}$$

$$\boxed{\begin{cases} C_1 e^{iqd} + D_1 e^{-iqd} = E_1 e^{ikd} \\ q(C_1 e^{iqd} - D_1 e^{-iqd}) = k E_1 e^{ikd} \end{cases}} \text{ where } \boxed{E_1 = E e^{ipd} e^{-ikd}} \quad (5)(6)$$

Eqs. (4), (5) are exactly the same as for the Right case, if we make the substitution $k \rightleftarrows p$.

Therefore, we can get the solutions immediately:

$$B_1 = \frac{i(q^2-pk)\sin qd + q(p-k)\cos qd}{-i(q^2+pk)\sin qd + q(k+p)\cos qd}, \quad \text{and } |B_1|^2 \equiv |B|^2$$

$$E_1 = \frac{2pq\, e^{-ikd}}{-i(q^2+kp)\sin qd + q(p+k)\cos qd}, \quad \text{and } |E_1|^2 = |E|^2.$$

~~where $k = \frac{1}{\hbar}\sqrt{2m\mathcal{E}}$, $q = \frac{1}{\hbar}\sqrt{2m(\mathcal{E}-U_0-V)}$, $p = \frac{1}{\hbar}\sqrt{2m(\mathcal{E}-V)}$ and so these are different (k and p are different), in fact, $k \rightleftarrows p$ to the case of $\rightarrow$. So,~~

~~$T_{\leftarrow} \equiv |E_2|^2 = |E_1|^2 = \left|\text{take the } \rightarrow \text{ expression and change } k \rightleftarrows p\right|$, the same for $R_{\leftarrow}$.~~

(i) $\mathcal{E} > U_0 + V$

$$\boxed{T_{\leftarrow} = \frac{4p^2q^2}{q^2(p+k)^2 + (q^2-p^2)(q^2-k^2)\sin^2(qd)} \equiv \frac{p^2}{k^2} T_{\rightarrow} \equiv |E|^2}$$

$$\boxed{R_{\leftarrow} = \frac{q^2(k-p)^2 + (q^2-p^2)(q^2-k^2)\sin^2(qd)}{q^2(k+p)^2 + (q^2-k^2)(q^2-p^2)\sin^2(qd)} \equiv R_{\rightarrow} \equiv |B|^2}$$

~~where we use our previous definitions for k, q, p~~ !

(ii) $V < \mathcal{E} < U_0 + V$

$$\boxed{T_{\leftarrow} = \frac{4p^2\lambda^2}{\lambda^2(p+k)^2 + (\lambda^2+p^2)(\lambda^2+k^2)\sinh^2(\lambda d)} \equiv \frac{p^2}{k^2} T_{\rightarrow}}$$

$$\boxed{R_{\leftarrow} = \frac{\lambda^2(k-p)^2 + (\lambda^2+k^2)(\lambda^2+p^2)\sinh^2(\lambda d)}{\lambda^2(k+p)^2 + (\lambda^2+k^2)(\lambda^2+p^2)\sinh^2(\lambda d)} \equiv R_{\rightarrow}}$$

~~Now we calculate the total cur~~

III Total current

The current in I is (positive — on the right!):

$$j_{I}^{tot} = \frac{\hbar k}{m}\left(1 - |B_{\rightarrow}|^2\right) - \frac{\hbar k}{m}|E_{\leftarrow}|^2 = \frac{\hbar k}{m}\left(1 - R_{\rightarrow} - T_{\leftarrow}\right)$$

The current in III is:

$$j_{III}^{tot} = \frac{\hbar p}{m}\left(|E_{\rightarrow}|^2 + |B_{\leftarrow}|^2 - 1\right) = \frac{\hbar p}{m}\left(T_{\rightarrow} + R_{\leftarrow} - 1\right)$$

We consider two energies:

(i) $\boxed{\mathcal{E} > U_0 + V}$

$$T_{\leftarrow} = \frac{4p^2q^2}{\mathcal{D}}, \quad T_{\rightarrow} = \frac{4k^2q^2}{\mathcal{D}}, \quad \mathcal{D} = q^2(k+p)^2 + (q^2-k^2)(q^2-p^2)\sin^2(qd)$$

$$R_{\leftarrow} = \frac{1}{\mathcal{D}}\left[q^2(k-p)^2 + (q^2-p^2)(q^2-k^2)\sin^2(qd)\right] \equiv R_{\rightarrow}$$

$$1 - R_{\rightarrow} - T_{\leftarrow} = \frac{1}{\mathcal{D}}\Big[q^2(k+p)^2 + \cancel{(q^2-k^2)(q^2-p^2)s^2} - q^2(k-p)^2 - \cancel{(q^2-p^2)(q^2-k^2)s^2}$$

$$- 4p^2q^2\Big] = \frac{q^2}{\mathcal{D}}\left[(k+p)^2 - (k-p)^2 - 4p^2\right] = \frac{q^2}{\mathcal{D}}\left(\cancel{k^2} + \cancel{p^2} + 2kp - \cancel{k^2} - \cancel{p^2} + 2kp - 4p^2\right)$$

$$= \frac{q^2}{\mathcal{D}}\left(4kp - 4p^2\right) = \frac{4pq^2}{\mathcal{D}}(k-p)$$

and hence

$$\boxed{j_{I}^{tot} = \frac{4\hbar q^2 kp}{m\mathcal{D}}(k-p)}$$

and it is the same as j_{III}^{tot}. Indeed,

$$T_{\rightarrow} + R_{\leftarrow} - 1 = \frac{1}{\mathcal{D}}\Big[4k^2q^2 + q^2(k-p)^2 + \cancel{(q^2-p^2)(q^2-k^2)s^2} - q^2(k+p)^2 -$$

$$- \cancel{(q^2-k^2)(q^2-p^2)s^2}\Big] = \frac{q^2}{\mathcal{D}}\left[4k^2 + (k-p)^2 - (k+p)^2\right] =$$

$$= \frac{q^2}{\mathcal{D}}\left(4k^2 + \cancel{k^2+p^2} - 2kp \; \cancel{-k^2-p^2} - 2kp\right) = \frac{q^2}{\mathcal{D}}\left(4k^2 - 4kp\right)$$

$$= \frac{4kq^2}{\mathcal{D}}(k-p), \quad \boxed{j_{III}^{tot} = \frac{\hbar p}{m}\frac{4kq^2}{\mathcal{D}}(k-p) \equiv j_I^{tot}}$$

(ii) $\boxed{V' < E < U_0 + V}$

$$T_{\rightarrow} = \frac{4k^2\lambda^2}{\mathcal{D}}, \quad T_{\leftarrow} = \frac{4p^2\lambda^2}{\mathcal{D}}, \quad \mathcal{D} = \lambda^2(p+k)^2 + (\lambda^2+p^2)(\lambda^2+k^2)\sinh^2(\lambda d)$$

$$R_{\rightarrow} = R_{\leftarrow} = \frac{1}{\mathcal{D}}\left[\lambda^2(k-p)^2 + (\lambda^2+k^2)(\lambda^2+p^2)\sinh^2(\lambda d)\right]$$

$$j_{III}^{tot} = \frac{\hbar p}{m}\left(T_{\rightarrow} + R_{\leftarrow} - 1\right) = \frac{\hbar p}{m\mathcal{D}}\left[4k^2\lambda^2 + \lambda^2(k-p)^2 + \cancel{(\lambda^2+k^2)(\lambda^2+p^2)s^2} -\right.$$

$$\left.- \lambda^2(p+k)^2 - \cancel{(\lambda^2+p^2)(\lambda^2+k^2)s^2}\right] = \frac{\hbar p\lambda^2}{m\mathcal{D}}\left[4k^2 + (k-p)^2 - (p+k)^2\right]$$

$$= \frac{\hbar p\lambda^2 4k(k-p)}{m\mathcal{D}}, \quad \boxed{j_{III}^{tot} = \frac{4\hbar\lambda^2 pk(k-p)}{m\mathcal{D}}}$$

and it is also the same in region I:

$$\cancel{j_I^{tot} = \frac{\hbar p}{m}\left(T_{\rightarrow} + R_{\leftarrow} - 1\right) = \frac{\hbar p}{m\mathcal{D}}\left[4k^2\lambda^2 + \lambda^2(k-p)^2 + (\lambda^2+k^2)(\lambda^2+p^2)s^2\right.}$$

$$j_I^{tot} = \frac{\hbar k}{m}\left(1 - R_{\rightarrow} - T_{\leftarrow}\right) = \frac{\hbar k}{m\mathcal{D}}\left[\lambda^2(p+k)^2 + \cancel{(\lambda^2+p^2)(\lambda^2+k^2)s^2} -\right.$$

$$\left.- \lambda^2(k-p)^2 - \cancel{(\lambda^2+k^2)(\lambda^2+p^2)s^2} - 4p^2\lambda^2\right] = \frac{\hbar k\lambda^2}{m\mathcal{D}}\left[(p+k)^2 - (k-p)^2 - 4p^2\right]$$

$$= \frac{\hbar k\lambda^2}{m\mathcal{D}} 4p(k-p)$$ which is indeed the same!

- Let us see that the current is $\emptyset$ if $V=0$: if $k=p$, then $j=0$. in both cases, as $j \sim k-p$.

• So, the summary of all equations:

$$\begin{cases} j^{tot}_{\rightarrow} = \dfrac{4\hbar}{m}\dfrac{q^2 kp}{\mathcal{D}}(k-p), \text{ if } \varepsilon > U_0 + V \\ \text{with } \mathcal{D} = q^2(k+p)^2 + (q^2-k^2)(q^2-p^2)\sin^2(qd) \\ \hline j^{tot}_{\rightarrow} = \dfrac{4\hbar}{m}\dfrac{\lambda^2 kp}{\mathcal{D}}(k-p), \text{ if } V < \varepsilon < U_0+V \\ \text{with } \mathcal{D} = \lambda^2(p+k)^2 + (\lambda^2+p^2)(\lambda^2+k^2)\sinh^2(\cancel{q}\lambda d) \end{cases}$$

where:

$$k = \frac{1}{\hbar}\sqrt{2m(\varepsilon - V)} \qquad q = \frac{1}{\hbar}\sqrt{2m(\varepsilon - U_0 - V)}$$

$$p = \frac{1}{\hbar}\sqrt{2m\varepsilon} \qquad \lambda = \frac{1}{\hbar}\sqrt{2m(U_0+V-\varepsilon)}$$

Let us simplify a bit by taking away the factor $\sqrt{2m}/\hbar$ from each wavevector; $\mathcal{D}$ is $\sim$ wavevector^4, so that this will cancell out:

$$\boxed{\begin{aligned} j^{tot}_{\rightarrow} &= \frac{4\hbar}{m}\frac{\bar{q}^2\bar{k}\bar{p}}{\bar{\mathcal{D}}}(\bar{k}-\bar{p})\frac{\sqrt{2m}}{\hbar}, \text{ if } \varepsilon > U_0+V \\ &= \frac{4\hbar}{m}\frac{\bar{\lambda}^2\bar{k}\bar{p}}{\bar{\mathcal{D}}}(\bar{k}-\bar{p})\frac{\sqrt{2m}}{\hbar}, \text{ if } V < \varepsilon < U_0+V \end{aligned}}$$

where $\bar{k} = \sqrt{\varepsilon - V}$, $\bar{q} = \sqrt{\varepsilon - U_0 - V}$, $\bar{\lambda} = \sqrt{U_0 + V - \varepsilon}$, $\bar{p} = \sqrt{\varepsilon}$

~~$\bar{\mathcal{D}} = \bar{q}^2(\bar{k}+\bar{p})^2 +$~~ and for $\varepsilon > U_0 + V$

$$\bar{\mathcal{D}} = (\varepsilon - U_0 - V)(\sqrt{\varepsilon - V} + \sqrt{\varepsilon})^2 + (\cancel{\varepsilon} - U_0 \cancel{-V} \cancel{-\varepsilon} \cancel{+V})(\cancel{\varepsilon} - U_0 - V \cancel{-\varepsilon})s^2$$

$$= (\varepsilon - U_0 - V)(\sqrt{\varepsilon} + \sqrt{\varepsilon - V})^2 + U_0(U_0+V)\sin^2(qd),$$

and for $V < \varepsilon < V + U_0$,

$$\bar{\mathcal{D}} = (U_0 + V - \varepsilon)(\sqrt{\varepsilon} + \sqrt{\varepsilon - V})^2 + (U_0 + V \cancel{-\varepsilon} \cancel{+\varepsilon})(U_0 \cancel{+V} \cancel{-\varepsilon} + \cancel{\varepsilon} \cancel{-V})\sinh^2(qd)$$

$$= (U_0 + V - \varepsilon)(\sqrt{\varepsilon} + \sqrt{\varepsilon - V})^2 + (U_0 + V)U_0 \sinh^2(\cancel{q}\lambda d)$$

So

$$j^{tot}_{\rightarrow} = \frac{4\sqrt{2}}{\sqrt{m}}\,\frac{\sqrt{\varepsilon-V}\sqrt{\varepsilon}}{\mathcal{D}}\left(\sqrt{\varepsilon-V}-\sqrt{\varepsilon}\right)\left(\varepsilon-U_0-V\right), \quad \underline{\varepsilon > U_0+V}$$

$$\mathcal{D} = |U_0+V-\varepsilon|\left(\sqrt{\varepsilon}+\sqrt{\varepsilon-V}\right)^2 + U_0(U_0+V)\sin^2(qd)$$

and for $\underline{V < \varepsilon < U_0+V}$

$$j^{tot}_{\rightarrow} = \frac{4\sqrt{2}}{\sqrt{m}}\,\frac{\sqrt{\varepsilon-V}\sqrt{\varepsilon}}{\mathcal{D}}\left(\sqrt{\varepsilon-V}-\sqrt{\varepsilon}\right)\left(U_0+V-\varepsilon\right)$$

$$\mathcal{D} = |U_0+V-\varepsilon|\left(\sqrt{\varepsilon}+\sqrt{\varepsilon-V}\right)^2 + U_0(U_0+V)\sinh^2(qd)$$

where $q = \frac{\sqrt{2m}}{\hbar}\sqrt{\varepsilon-U_0-V}$ and $\lambda = \frac{\sqrt{2m}}{\hbar}\sqrt{U_0+V-\varepsilon}$.

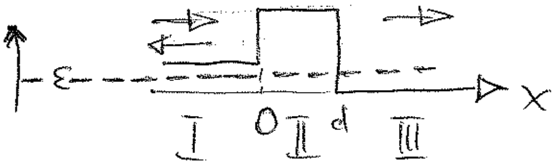
These are the final expressions..(electron charge is missing).

- The conductivity is obtained by

$$\sigma = \left.\frac{\partial(j\cdot e)}{\partial V}\right|_{V\to 0}, \quad e - \text{electron charge.}$$

IV Energies $0 < \varepsilon < V$

(A) going to the right



$$\psi_I = Ae^{ikx} + Be^{-ikx}$$ (both should be kept up to now)

$$\psi_{II} = Ce^{iqx} + De^{-iqx}$$

$$\psi_{III} = E\,e^{ipx} + F e^{-ipx}$$

where

$$k = \frac{1}{\hbar}\sqrt{2m(\varepsilon - V)} = i\varkappa, \quad \varkappa = \frac{1}{\hbar}\sqrt{2m(V-\varepsilon)}$$

$$q = i\lambda, \quad \lambda = \frac{1}{\hbar}\sqrt{2m(U_0 + V - \varepsilon)}$$

$$p = \frac{1}{\hbar}\sqrt{2m\varepsilon} \quad \text{(real)}$$

Here $\psi_I = Ae^{-\varkappa x} + Be^{\varkappa x} \Rightarrow Be^{\varkappa x}$, since at $x = -\infty$ the 1st term grows indefinitely.

$$\psi_{II} = Ce^{-\lambda x} + De^{\lambda x}, \quad \psi_{III} = E\,e^{ipx}$$

Continuity at $x = 0$:

$$\begin{cases} B = C + D \\ B\varkappa = \lambda(-C + D) \end{cases}$$

$$\varkappa(C+D) = -\lambda(C - D)$$

$$C(\varkappa + \lambda) = (\lambda - \varkappa)D$$

$$\boxed{C = \frac{\lambda - \varkappa}{\lambda + \varkappa} D} \longrightarrow$$

Continuity at $x = d$

$$\begin{cases} Ce^{-\lambda d} + De^{\lambda d} = E\,e^{ipd} + F e^{-ipd} \\ \lambda(-Ce^{-\lambda d} + De^{\lambda d}) = ip(E\,e^{ipd} - F e^{-ipd}) \end{cases}$$

$$\cancel{(Ce^{-\lambda d} + De^{\lambda d})\,ip = -\lambda(Ce^{-\lambda d} - De^{\lambda d})}$$

$$\cancel{\left(\frac{\lambda-\varkappa}{\lambda+\varkappa}e^{-\lambda d} + e^{\lambda d}\right) ip = -\lambda\left(\frac{\lambda-\varkappa}{\lambda+\varkappa}e^{-\lambda d} - e^{\lambda d}\right)}$$

~~This is an equation for discrete states.~~

$$\left.\begin{aligned} &\left(\frac{\lambda-\varkappa}{\lambda+\varkappa}e^{-\lambda d} + e^{\lambda d}\right)D = e^{-ipd} + Ee^{ipd} \\ &\left(-\frac{\lambda-\varkappa}{\lambda+\varkappa}e^{-\lambda d} + e^{\lambda d}\right)D = \left[-e^{-ipd} + Ee^{ipd}\right]\frac{ip}{\lambda} \end{aligned}\right\}$$

$$\left(-\frac{\lambda-\varkappa}{\lambda+\varkappa}e^{-\lambda d} + e^{\lambda d}\right)D = \frac{-ip}{\lambda}e^{-ipd} + \frac{ip}{\lambda}\left\{-e^{-ipd} + \left(\frac{\lambda-\varkappa}{\lambda+\varkappa}e^{-\lambda d} + e^{\lambda d}\right)D\right\}$$

$$\left[\left(-\frac{\lambda-\varkappa}{\lambda+\varkappa}e^{-\lambda d} + e^{\lambda d}\right) - \frac{ip}{\lambda}\left(\frac{\lambda-\varkappa}{\lambda+\varkappa}e^{-\lambda d} + e^{\lambda d}\right)\right]D = -\frac{2ip}{\lambda}e^{-ipd}$$

$$[\ldots] = \frac{\lambda-\varkappa}{\lambda+\varkappa} e^{-\lambda d}\left(-1-\frac{ip}{\lambda}\right) + e^{\lambda d}\left(1-\frac{ip}{\lambda}\right)$$

$$= -\frac{(\lambda+ip)(\lambda-\varkappa)}{\lambda(\lambda+\varkappa)} e^{-\lambda d} + \frac{\lambda-ip}{\lambda} e^{\lambda d}$$

$$= \frac{1}{\lambda(\lambda+\varkappa)}\left[-(\lambda+ip)(\lambda-\varkappa)e^{-\lambda d} + (\lambda-ip)(\lambda+\varkappa)e^{\lambda d}\right]$$

$$D = \frac{-2ip\, e^{-ipd}\, \cancel{\lambda}(\lambda+\varkappa)}{\cancel{\lambda}\left[(\lambda-ip)(\lambda+\varkappa)e^{\lambda d} - (\lambda+ip)(\lambda-\varkappa)e^{-\lambda d}\right]}$$

$$\boxed{D = \frac{-2ip(\lambda+\varkappa)\, e^{-ipd}}{(\lambda-ip)(\lambda+\varkappa)e^{\lambda d} - (\lambda+ip)(\lambda-\varkappa)e^{-\lambda d}}}$$

is complex in general.

$$B = C + D = \left(\frac{\lambda-\varkappa}{\lambda+\varkappa}+1\right) D = \frac{2\lambda}{\lambda+\varkappa} D$$

$$\boxed{B = \frac{-4ip\, e^{-ipd}}{(\lambda-ip)(\lambda+\varkappa)e^{\lambda d} - (\lambda+ip)(\lambda-\varkappa)e^{-\lambda d}}}$$

$$E e^{ipd} + e^{-ipd} = C e^{-\lambda d} + D e^{\lambda d} = \left(\frac{\lambda-\varkappa}{\lambda+\varkappa} e^{-\lambda d} + e^{\lambda d}\right) D$$

$$= \frac{(\lambda-\varkappa)e^{-\lambda d} + (\lambda+\varkappa)e^{\lambda d}}{\cancel{\lambda+\varkappa}} \cdot \frac{-2ip\,\cancel{(\lambda+\varkappa)}\, e^{-ipd}}{\ldots}$$

$$= -2ip\, e^{-ipd} \frac{(\lambda-\varkappa)e^{-\lambda d} + (\lambda+\varkappa)e^{\lambda d}}{(\lambda-ip)(\lambda+\varkappa)e^{\lambda d} - (\lambda+ip)(\lambda-\varkappa)e^{-\lambda d}}$$

$$E = -e^{-2ipd} - 2ip\, e^{-2ipd} \frac{\ldots}{}$$

$$E e^{2ipd} = -1 - \frac{2ip(\lambda-\varkappa)e^{-\lambda d} + 2ip(\lambda+\varkappa)e^{\lambda d}}{(\lambda-ip)(\lambda+\varkappa)e^{\lambda d} - (\lambda+ip)(\lambda-\varkappa)e^{-\lambda d}} = \frac{E_1}{E_2}$$

$$E_1 = -(\lambda-ip)(\lambda+\varkappa)e^{\lambda d} + (\lambda+ip)(\lambda-\varkappa)e^{-\lambda d} - 2ip(\lambda-\varkappa)e^{-\lambda d} - 2ip(\lambda+\varkappa)e^{\lambda d}$$

$$= e^{\lambda d}(\lambda+\varkappa)(-\lambda+ip-2ip) + (\lambda-\varkappa)e^{-\lambda d}(\lambda+ip-2ip)$$

$$= -e^{\lambda d}(\lambda+\varkappa)(+\lambda+ip) + (\lambda-\varkappa)(\lambda-ip)e^{-\lambda d}$$

$$\boxed{E_{3} = \frac{-(\lambda+\varkappa)(\lambda+ip)e^{\lambda d} + (\lambda-\varkappa)(\lambda-ip)e^{-\lambda d}}{(\lambda+\varkappa)(\lambda-ip)e^{\lambda d} - (\lambda-\varkappa)(\lambda+ip)e^{-\lambda d}} \cdot e^{-2ipd}}$$

All ε are eigenvalues. Only one solution exists for each $0<\varepsilon<V$.

The current: $j = \frac{i\hbar}{2m}(\psi\psi^{*\prime} - \psi^*\psi^\prime)$

$$\psi_I = Be^{\varkappa x}, \quad \psi_I^\prime = \varkappa B e^{\varkappa x}$$

$$j_I = \frac{i\hbar}{2m}\left[Be^{\varkappa x}\varkappa B^* e^{\varkappa x} - B^* e^{\varkappa x}\varkappa B e^{\varkappa x}\right] = 0$$

Let us calculate j_{III}:

$$\cancel{j_{III} = \frac{\hbar p}{m}(|E|^2 - 1)} \qquad \psi_{III} = e^{-ipx} + Ee^{ipx}$$

$$\psi_{III}^\prime = -ip\,e^{-ipx} + ipE\,e^{ipx}$$

$$j_{III} = \frac{i\hbar}{2m}\Big\{(e^{-ipx} + Ee^{ipx})\,ip\,(e^{ipx} - E^*e^{-ipx})$$

$$- (e^{+ipx} + E^*e^{-ipx})\,ip\,(-e^{-ipx} + Ee^{ipx})\Big\}$$

$$= \frac{i\hbar}{2m}ip\Big\{1 + \cancel{Ee^{2ipx}} - \cancel{E^*e^{-i2px}} - |E|^2 - (-1 - \cancel{E^*e^{-i2px}} + \cancel{Ee^{i2px}}$$

$$+|E|^2)\Big\} = \frac{-\hbar p}{2m}\{2 - 2|E|^2\} = \frac{\hbar p}{m}(|E|^2 - 1)$$

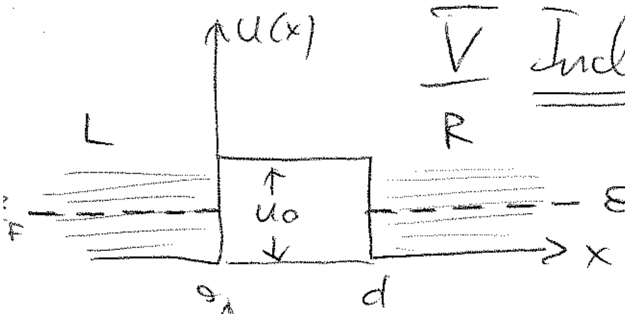
$$|E|^2 = \frac{\left[(\lambda-\varkappa)\lambda e^{-\lambda d} - \lambda(\lambda+\varkappa)e^{\lambda d}\right]^2 + \left[p(\lambda+\varkappa)e^{\lambda d} - p(\lambda-\varkappa)e^{-\lambda d}\right]^2}{\left[\lambda(\lambda+\varkappa)e^{\lambda d} - \lambda(\lambda-\varkappa)e^{-\lambda d}\right]^2 + \left[p(\lambda+\varkappa)e^{\lambda d} - p(\lambda-\varkappa)e^{-\lambda d}\right]^2} \equiv 1$$

and hence $j_{III} = 0$ as well.

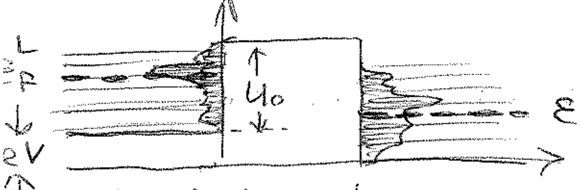
Therefore, states $0 < \varepsilon < V$ do not contribute to the conductivity.

$\underline{V}$ Inclusion of Fermi sea $\boxed{\text{STM}}$

$\boxed{\text{Assumed } V<0}$



ε_F ← no bias applied.



$\varepsilon_F^R \equiv \varepsilon_F$ ← if bias V is applied.

All electron states in the left lead shift by eV (up by $|eV|$ if $V<0$), so that $\boxed{\varepsilon_F^L = \varepsilon_F^R + eV} \equiv \varepsilon_F + eV$

The current to the right of the state of energy ε is given, e.g. by

$$j^{\rightarrow}(\varepsilon) = \frac{\hbar p}{m} T_{\rightarrow}(\varepsilon), \quad \text{where } \varepsilon > eV, \cancel{\text{and } < \varepsilon}, \cancel{\text{and the DOS}}$$

and the total current from all available states is:

$$j_{tot}^{\rightarrow} = \sum_{\varepsilon > eV} \frac{\hbar p}{m} T_{\rightarrow}(\varepsilon) f_L(\varepsilon), \text{ where } f(\varepsilon) = \frac{1}{e^{-\frac{\hbar(\varepsilon_F^L - \varepsilon)}{kT}} + 1} = \left[e^{\beta(\varepsilon - \varepsilon_F^L)} + 1\right]^{-1}$$

If $\mathcal{D}_L(\varepsilon)$ is the electron DOS, then

$$j_{\rightarrow}^{tot} = \int_{eV}^{\infty} \frac{\hbar p}{m} T_{\rightarrow}(\varepsilon) f_L(\varepsilon) \underbrace{\mathcal{D}_L(\varepsilon - eV)}_{\text{DOS "moves" together with all states}} d\varepsilon \qquad (1)$$

Here

$$T_{\rightarrow}(\varepsilon) = \frac{4k^2\lambda^2}{\lambda^2(p+k)^2 + (\lambda^2+p^2)(\lambda^2+k^2)\sinh^2(\lambda d)}, \quad \text{if } eV < \varepsilon < U_0 + eV$$

$$= \frac{4k^2q^2}{q^2(p+k)^2 + (q^2-p^2)(q^2-k^2)\sin^2(qd)}, \quad \text{if } \varepsilon > U_0 + eV$$

and $k = \frac{1}{\hbar}\sqrt{2m(\varepsilon - eV)}$, $q = \frac{1}{\hbar}\sqrt{2m(\varepsilon - U_0 - eV)}$, $\lambda = \frac{1}{\hbar}\sqrt{2m(U_0 + eV - \varepsilon)}$

$p = \frac{1}{\hbar}\sqrt{2m\varepsilon}$

Also $f_L(\varepsilon) = \left[e^{\beta(\varepsilon - \varepsilon_F - eV)} + 1\right]^{-1} \equiv f_0(\varepsilon - eV)$, where

$f_0(\varepsilon) = \left[e^{\beta(\varepsilon - \varepsilon_F)} + 1\right]^{-1}$ is usual Fermi-Dirac distribution

Hence,

$$j^{tot}_{\rightarrow} = \int_{eV}^{\infty} \frac{\hbar p}{m} T_{\rightarrow}(\varepsilon) f_0(\varepsilon - eV) \mathcal{D}_L(\varepsilon - eV)\, d\varepsilon$$

with

$$T_{\rightarrow}(\varepsilon) = \begin{cases} \dfrac{4(\varepsilon-eV)(U_0+eV-\varepsilon)}{(U_0+eV-\varepsilon)(\sqrt{\varepsilon}+\sqrt{\varepsilon-eV})^2+U_0(U_0+eV)\sinh^2(\lambda d)}, & eV<\varepsilon<U_0+eV \\ \dfrac{4(\varepsilon-eV)(\varepsilon-U_0-eV)}{(\varepsilon-U_0-eV)(\sqrt{\varepsilon}+\sqrt{\varepsilon-eV})^2+U_0(U_0+eV)\sin^2(qd)}, & \varepsilon>U_0+eV \end{cases}$$

$$\frac{\hbar p}{m} = \frac{\sqrt{2m\varepsilon}}{m} = \sqrt{\frac{2\varepsilon}{m}} \sim \sqrt{\varepsilon}, \quad f_0(\varepsilon) = \left[e^{\beta(\varepsilon-\varepsilon_F)}+1\right]^{-1}.$$

Basically, for zero T, $f_0(\varepsilon) = \theta(\varepsilon_F - \varepsilon)$, so that only states $\varepsilon - eV \le \varepsilon_F$, $\varepsilon \le \varepsilon_F + eV$ will contribute, OK!

- The Right-to-Left current due to state ε

$$j_{\leftarrow}(\varepsilon) = \frac{\hbar p}{m}(R_{\leftarrow} - 1)$$

and, due to all states,

$$j^{tot}_{\leftarrow} = \sum_{\varepsilon > eV} \frac{\hbar p}{m}(R_{\leftarrow}-1) f_R(\varepsilon) \rightarrow \int_{eV}^{\infty} \frac{\hbar p}{m}(R_{\leftarrow}-1) f_0(\varepsilon) D_R(\varepsilon)\, d\varepsilon$$

since $f_R \equiv f_0$. Here,

$$R_{\leftarrow}(\varepsilon) = R_{\rightarrow}(\varepsilon) = \begin{cases} \dfrac{q^2(k-p)^2+(q^2-p^2)(q^2-k^2)\sin^2(qd)}{q^2(k+p)^2+(q^2-p^2)(q^2-k^2)\sin^2(qd)}, & \varepsilon > U_0+eV \\ \dfrac{\lambda^2(k-p)^2+(\lambda^2+k^2)(\lambda^2+p^2)\sinh^2(\lambda d)}{\lambda^2(k+p)^2+(\lambda^2+k^2)(\lambda^2+p^2)\sinh^2(\lambda d)}, & eV<\varepsilon<U_0+eV \end{cases}$$

We calculate $R_{\leftarrow} - 1$. For $eV < \varepsilon < U_0 + eV$,

$$R_{\leftarrow} - 1 = \frac{1}{D_n}\left\{\lambda^2(k-p)^2 + \cancel{(\lambda^2+k^2)(\lambda^2+p^2)S^2} - \lambda^2(k+p)^2 - \cancel{(\lambda^2+k^2)(\lambda^2+p^2)S^2}\right\}$$

$$= \frac{\lambda^2}{D_n}(\cancel{k}-p-\cancel{k}-p)(k-\cancel{p}+k+\cancel{p}) = \frac{\lambda^2}{\mathcal{D}_n}2k(-2p) = -\frac{4kp\lambda^2}{\mathcal{D}_n} =$$

$$= -\frac{\cancel{4}kp\cancel{\lambda^2}}{\cancel{4}p^2\cancel{\lambda^2}} \cdot \frac{4p^2\lambda^2}{\underbrace{\mathcal{D}_A}_{T_\leftarrow}} = -\frac{k}{p}T_\leftarrow = -\frac{k}{p}\frac{p^2}{k^2}T_\rightarrow = -\frac{p}{k}T_\rightarrow$$

i.e. $\boxed{R_\leftarrow - 1 = -\frac{p}{k}T_\rightarrow}$ for this energy range. $\boxed{R_\leftarrow - 1 = -\frac{k}{p}T_\leftarrow = -\frac{p}{k}T_\rightarrow}$

Consider now $\underline{\varepsilon > U_0 + eV}$:

$$R_\leftarrow - 1 = \frac{q^2}{\mathcal{D}_n}\left\{q^2(k-p)^2 + (q^2-p^2)\cancel{(q^2-k^2)}s^2 - q^2(k+p)^2 - \cancel{(q^2-k^2)(q^2-p^2)s^2}\right\}$$

$$= \frac{q^2}{\mathcal{D}_n}(\cancel{k}-p-\cancel{k}-p)(k-\cancel{p}+k+\cancel{p}) = -\frac{4kpq^2}{\mathcal{D}_n} = -\frac{4kp\cancel{q^2}}{4p^2\cancel{q^2}}\,\underbrace{\frac{4p^2q^2}{\mathcal{D}_n}}_{T_\leftarrow}$$

$$= -\frac{k}{p}T_\leftarrow = -\frac{k}{p}\frac{p^2}{k}T_\rightarrow = -\frac{p}{k}T_\rightarrow$$, the same. So, this is general for any energy $\varepsilon > eV$. Consequently,

$$j_\leftarrow^{tot} = \int_{eV}^{\infty}\frac{\hbar p}{m}\left(-\frac{p}{k}T_\rightarrow\right)f_0(\varepsilon)\mathcal{D}_R(\varepsilon)\,d\varepsilon$$

- The total current is the sum of the two:

$$\boxed{j_{tot} = \int_{eV}^{\infty}\frac{e\hbar p}{m}\left[f_0(\varepsilon-eV)\mathcal{D}_L(\varepsilon-eV) - \frac{p}{k}f_0(\varepsilon)\mathcal{D}_R(\varepsilon)\right]T_\rightarrow(\varepsilon)\,d\varepsilon}$$

~~Assuming uniform DOS, we set~~

Alternative form:

$$\boxed{j_{tot} = \frac{e\hbar}{m}\int_{eV}^{\infty}\left[p f_0(\varepsilon-eV)\mathcal{D}_L(\varepsilon-eV)T_\rightarrow(\varepsilon) - k f_0(\varepsilon)\mathcal{D}_R(\varepsilon)T_\leftarrow(\varepsilon)\right]d\varepsilon}$$

with $p = \frac{1}{\hbar}\sqrt{2m\varepsilon}$, $k = \frac{1}{\hbar}\sqrt{2m(\varepsilon - eV)}$