Worksheet 11

There is a quiz of basic understanding for lecture 11 on Keats.

Solutions to the exercises below can be found in the file `solutions.pdf` in `lecture11.zip`

1) [⋆] A stock follows the Black Scholes model with $S_0 = 1$, $\sigma = 0.2$ and $\mu = 0.08$. The risk free rate is 0.05. An investor has 1 dollar to invest for a time period of 1 year and wishes to optimize their expected utility. Their utility function is

$$u(x) = \begin{cases} \ln(x) & x > 0 \\ -\infty & x \leq 0 \end{cases}$$

Compute their expected utility:

(a) By the Monte Carlo method

(b) By the Monte Carlo method with antithetic sampling

(c) By the Monte Carlo method with a control variate of your choice

(d) Using a low discrepancy sequence.

(e) Use the rectangle rule.

(f) Compare the errors of these approaches

2) [⋆] You can compute the area of the unit circle using a Monte Carlo method. Simply generate uniformly distributed points in $[-1, 1] \times [1, 1]$ and count how many lie in the circle. Implement this in MATLAB.

Which is better using $2N$ uniformly distributed points in $[-1, 1] \times [-1, 1]$ or using $2N$ points generated using antithetic sampling? Explain your answer.

3) [⋆] Use Richardson extrapolation to improve the estimator

$$f'(x) \approx \frac{f(x + h) - f(x)}{h}$$

Generate log-log plots to illustrate your solution.

4) [⋆] Do the May 2016 exam in its entirety