King's College London

University Of London

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MSc Examination

7CCMFM06 Numerical and Computational Methods in Finance

Summer 2015

Time Allowed: Two Hours

All questions carry equal marks. Full marks will be awarded for complete answers to FOUR questions. Only the best FOUR questions will count towards grades A or B, but credit will be given for all work done for lower grades.

NO calculators are permitted.

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TURN OVER WHEN INSTRUCTED

1. Let S be the region of R^2 defined by the equation

$$x^4 + y^4 \leq 1$$

One can approximate the area of S using a Monte Carlo algorithm as follows: generate a large number, N, of random, uniformly distributed points in the square $[-1, 1] \times [-1, 1]$; calculate the proportion of these points that lie in the square to obtain an estimate for S.

(a) Write a MATLAB function to implement this algorithm. [**50%**]

(b) How could you test your function? [**10%**]

(c) Suggest an alternative algorithm with better convergence. State the rate of convergence of both algorithms. [**20%**]

(d) Would the use of antithetic sampling improve the convergence of the Monte Carlo algorithm? Justify your answer. [**20%**]

2. (a) Describe the delta hedge trading strategy. [**10%**]

(b) Describe the gamma hedge trading strategy. [**10%**]

(c) A trader writes an option O^1 for a price P and gamma hedges at fixed time points $(0, \delta t, 2\delta t, \ldots, N\delta t)$ using an option O^2. Derive an expression for the quantity of stock and the quantity of option O^2 held at each time point in terms of the delta and gamma of each option. [**20%**]

(d) Derive a difference equation for the trader's bank balance at each time point before maturity assuming that it grows at a risk free rate r between time steps. [**20%**]

(e) Draw a plot which indicates the sense in which the delta and gamma hedging strategies "converge" as δt tends to zero. Be sure to explain what the axes of your graph are and how the plot should be interpreted. Show how you would expect the plot to change when transaction costs are included. [**20%**]

(f) With reference to your plots, explain the financial motivation for gamma hedging. [**20%**]

3. (a) A stock price S^1 follows the stochastic differential equation:

$$\mathrm{d}S^1_t = S^1_t(r\,\mathrm{d}t + \sigma_1\,\mathrm{d}W^1_t)$$

in the risk neutral measure. W^1 is a Brownian motion.

Describe how you could simulate prices S^1_T at some future time T in the risk neutral measure? **[20%]**

(b) A second stock price S^2 follows the stochastic differential equation:

$$\mathrm{d}S^2_t = S^2_t(r\,\mathrm{d}t + \sigma_2\,\mathrm{d}W^2_t)$$

Again W^2 is a Brownian motion and we are working in the risk neutral measure. However W^1 and W^2 are correlated with correlation ρ.

(i) Define the Cholesky decomposition of a positive definite symmetric matrix B. **[10%]**

(ii) Find the Cholesky decomposition of

$$\begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix}$$

[20%]

(iii) Use this to describe how you could simulate pairs of prices (S^1_T, S^2_T) at some future time T. **[20%]**

(iv) How could you use this simulation to compute the risk neutral price of a derivative which pays $\max\{S^1_T - K^1, S^2_T - K^2, 0\}$ at maturity T for some fixed strike prices K^1 and K^2? **[10%]**

(v) If you were to implement this in MATLAB, what functions you would write and how you would test them? There is no need to implement the functions. You should simply provide good names for the functions, describe briefly what each function does and suggest how they could be tested. **[20%]**

4. (a) Let $f : [a, b] \longrightarrow \mathrm{R}$ be a smooth function.

(i) Name two numerical methods that could be applied to compute the integral

$$\int_a^b f(x)\,\mathrm{d}x$$

and state their rates of convergence. [**10%**]

(ii) Write a MATLAB function to implement one of these numerical methods. You may assume that the integrand is passed as a parameter to your function. [**30%**]

(b) A trader's current portfolio consists of a short position in a call option on a stock S with strike K and maturity T and a risk free account with balance B.

The trader believes that the stock price follows the Black–Scholes model:

$$\mathrm{d}S_t = S_t(\mu\,\mathrm{d}t + \sigma\,\mathrm{d}W_t).$$

The risk free account has interest rate r.

The trader intends to buy a certain quantity, q, of stock at time 0 and then hold it until time T. They wish to choose the amount of stock to purchase in order to maximize their expected utility at time T. Their utility at time T is given by applying some smoooth, concave real valued function u to the value of their portfolio at time T.

(i) Assuming the trader purchases q units of stock, give an explicit expression for the expected utility at time T as an integral. [**40%**]

(ii) If necessary show how this integral can be rewritten so it has finite limits of integration [**10%**]

(iii) How could you use MATLAB to find the optimum value of q? [**10%**]

5. The Black–Scholes p.d.e. is

$$\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + rS\frac{\partial V}{\partial S} - rV = 0$$

(a) Make the change of variable $z = \log S$ and write the resulting p.d.e. for V as a function of z and t. What are the benefits, if any, of the change of variables? [**30%**]

(b) We wish to apply the explicit finite difference method to the transformed equation to price a European put option with strike K and maturity T.

(i) Derive the difference equation for the explicit method. [**30%**]

(ii) Over what region of the (z, t) plane would you define your grid? Explain your answer. [**20%**]

(iii) What boundary conditions would you use for the p.d.e.? [**20%**]