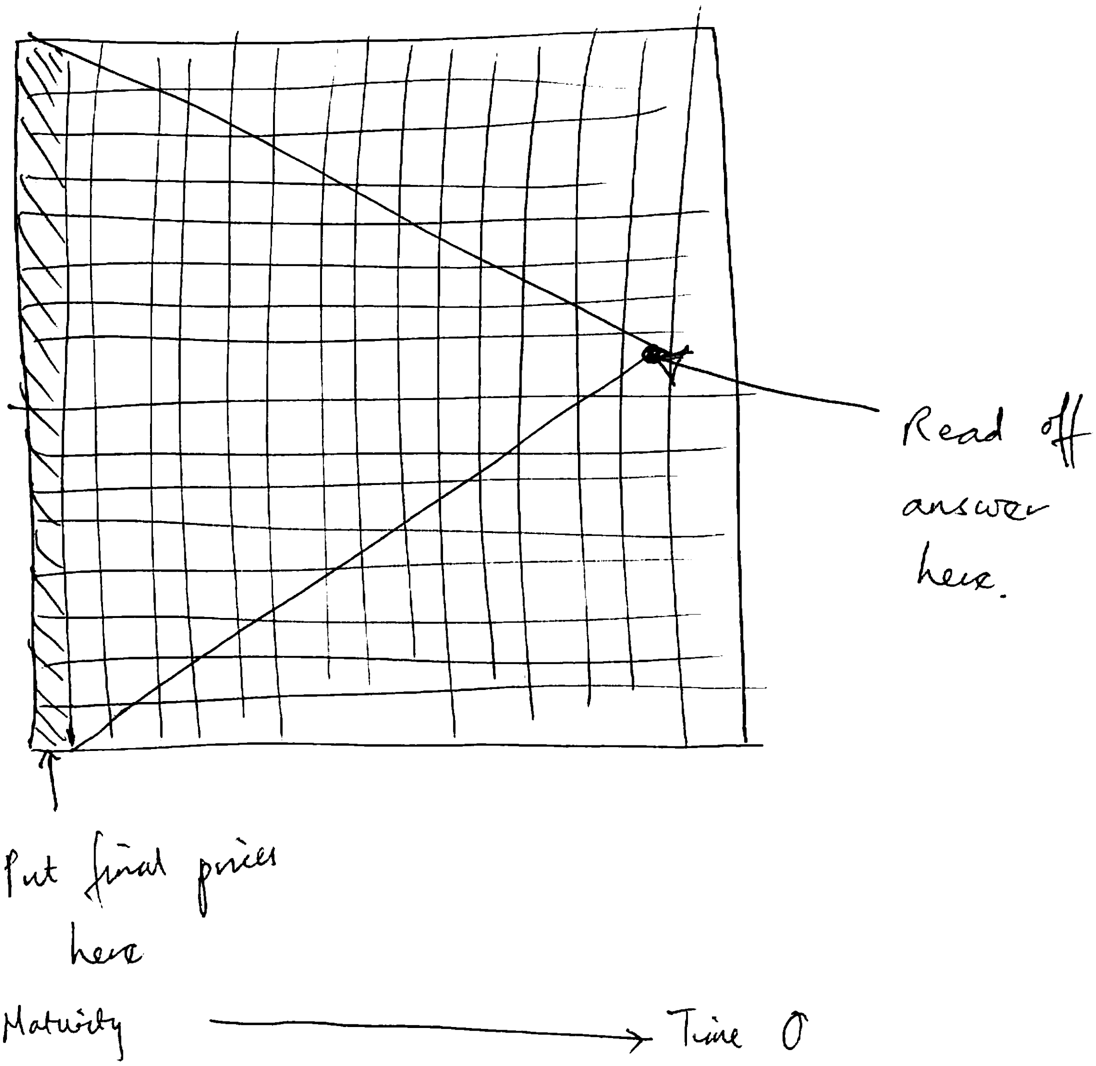
TRINOMIAL TREE HINTS

To create a trinomial tree price in Excel you just need to create a large spreadsheet:



The larger the spreadsheet, the more accurate the answer! In practice Excel can cope with about 500 time steps and this gives a reasonable approximation.

In a trinomial tree price you assume that from time t to time t+1, the stock price can move up (u), down (d) or stay the same (m). If the stock price was S then if it moves up the new stock price is uS. If it moves down the new stock price is dS.

The RISK NEUTRAL probability of it moving up, down or staying the same are p_u, p_d and p_m respectively.

So there are 6 parameters to choose for your trinomial tree: N, u, d, p_u, p_d, p_m. N is the number of steps. So each time step is $\Delta t = T/N$.

There are various constraints on these parameters.

(1) $d = 1/u$ — i.e. the tree is "recombining"

(2) $1 = p_u + p_d + p_m$

(3) No abitrage: $\mathbb{E}(S(t+\Delta t) \mid S(t)) = e^{r\Delta t} S(t)$

$$\cancel{1-p_u-p_d}\; p_m + p_u u + p_d d = e^{r\Delta t}$$

(4) Moment matching:

$$\mathrm{Var}(S(t+\Delta t) \mid S(t)) = \Delta t\, S(t)^2 \sigma^2 + O(\Delta t)$$

This means we are free to choose N and u, but then our 4 constraints determine the other 4 parameters. However, not all choices of u work: we need the tree to converge. ~~and~~ One choice that works is:

$$u = e^{\sigma\sqrt{2\Delta t}}$$

$$p_u = \left(\frac{e^{\frac{r\Delta t}{2}} - e^{-\sigma\sqrt{\frac{\Delta t}{2}}}}{e^{\sigma\sqrt{\frac{\Delta t}{2}}} - e^{-\sigma\sqrt{\frac{\Delta t}{2}}}} \right)^2$$

$$p_d = \left(\frac{e^{\sigma\sqrt{\frac{\Delta t}{2}}} - e^{\frac{r\Delta t}{2}}}{e^{\sigma\sqrt{\frac{\Delta t}{2}}} - e^{-\sigma\sqrt{\frac{\Delta t}{2}}}} \right)^2$$

You should verify that this meets all the constraints (don't blame me if I've made a slip of the pen!)

Since p_u, p_d, p_m are risk neutral probabilities you can now compute the value of an option at time t using the discounted expected value at time $t + \Delta t$. You know the final payoff of an option, so by a backwards induction you can compute the initial price.

- Use this to price a European option and compare it with Black-Scholes
- ~~At each point~~ To price an American option, you compute the maximum of the expected price at time $t + \Delta t$ and the value obtained if you exercise at time t. So with only a small modification you can price American options.

<u>Some more ideas</u>

- Have you any idea how the price of American options differs from European options? Now's your chance to find out and tell us all.
- Its a fact that if interest rates are non negative then European and American call options on a non-dividend paying stock have the same value. Can you prove this / find a proof in the literature?

- Dividends clearly affect the value of American options. Can you include them in you pricer? (This is probably quite hard, but would be cool)
- Can you price knock out options?