

## OPTIMIZATION HINTS

Suppose that you believe the stock price will follow the B-S model with mean  $\mu$  and volatility  $\sigma$ .

You already know how to generate 10000 future stock prices.

• So you can write a function to ~~price~~ <sup>compute the</sup> payoff of a portfolio of options.

• Represent the ~~vector~~ portfolio as ~~an~~  $2n$  dimensional vector,  $\underline{v}$ . Let the  $(2i-1)^{\text{th}}$  entry of  $\underline{v}$  be the quantity of option  $i$  that you have bought and let the  $(2i)^{\text{th}}$  entry of  $\underline{v}$  be the quantity of option  $i$  that you have sold.

• Let  $\underline{c}$  be a vector representing the initial cost. Its  $(2i-1)^{\text{th}}$  and  $(2i)^{\text{th}}$  entries are given by the ask and bid prices. So your initial cost is  $\underline{c} \cdot \underline{v}$ .

- You can compute a payoff matrix  $P$  such that the vector

$$P \underline{v}$$

consists of the payoffs for each of your 10000 scenarios.

- Find the choice of  $\underline{v}$  which optimizes the <sup>expected</sup> utility

$$E(u(p))$$

for the utility function

$$u(p) = 1 - \exp(-p \lambda)$$

where  $\lambda$  is a risk aversion parameter.

The constraints are:

$$v_j > 0$$

$$\text{and } \underline{c} \cdot \underline{v} = 1 \quad (\text{you have an}$$

initial budget of \$1)

- Compute  $\underline{v}$  using real data for option and stock prices. You can choose your own values for  $\mu$ ,  $\sigma$  and  $\lambda$ . These represent

your beliefs about the market and your risk appetite.

- When you've computed the optimal portfolio  $v$ , plot a scatter plot of its payoff against the final stock price.
- You should ~~include positions of~~ ~~on the "options"~~ allow the portfolio to include purchases of the stocks and ~~low~~ zero coupon bonds too.
- You can see what happens if you add the additional constraint of disallowing short selling.

To perform optimization you will need

to use the ~~function~~ `fmincon` function

in MATLAB. I suggest you start by

getting an understanding of this function.

One complexity is that `fmincon`

takes a function as an argument.

You can define a function in the

following way in MATLAB

`myfunction = @(x) x^2 + 2x - 2;`

`minx = fmincon(myfunction, 0);`

`disp(minx);`

This minimizes the function  $x^2 + 2x - 2$ .

Good luck!