GEMSTONE mini-course: Variations of Geometry and Spectrum

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May 9, 2025: Continuity

A wonderful thing about eigenvalues and eigenfunctions is that they depend continuously on the geometry of the domain. In fact, this continuity property is so robust that it sometimes even holds when the topology changes. In this lecture, I will explain what underlies continuity and describe a few of the myriad applications.

May 12, 2025: Analyticity

If the geometry varies analytically, then the eigenvalues and eigenfunctions will also vary analytically (with caveats). Because an analytic function is globally determined by its local behavior, this is a very potent property. I will describe the analyticity property and its caveats with applications to the multiplicity problem and, if time permits, the existence of cusp forms.

May 13, 2025: Transversality

In the 70's, Vladimir Arnol'd and Karen Uhlenbeck independently introduced transversality into spectral geometry. For example, Uhlenbeck used it to show that in some basic contexts the set of triples (eigenvalue, eigenvector, domain) has a natural manifold structure. In this lecture I will describe the basic idea of parametric transversality, its original application by Uhlenbeck, and how it continues to serve us.