
ON A CONJECTURE OF A PÓLYA FUNCTIONAL FOR TRIANGLES AND RECTANGLES

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We consider a functional involving the product of the first Dirichlet eigenvalue $\lambda_1(\Omega)$ and the torsional rigidity $T(\Omega)$ of a planar domain Ω normalized by its area $|\Omega|$. This functional was originally considered by Pólya who first showed that this quantity is bounded by 1. It has been conjectured that this functional is bounded above by $\pi^2/12$ and below by $\pi^2/24$ over the class of bounded planar convex domains. We prove this is true for the class of all triangles and rectangles. In particular, we prove precise estimates for triangles and a monotonicity property for rectangles. This talk is based on joint work with Rodrigo Bañuelos.