Small eigenvalues of many cusped hyperbolic surfaces

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Based on joint work with Joe Thomas (Durham)

We study the spectrum of the Laplacian for non-compact finite-area hyperbolic surfaces, focusing on eigenvalues below $\frac{1}{4}$ which are known as small eigenvalues. A result of Otal and Rosas says that any hyperbolic surface of genus g with n cusps has at most $2g + n - 2$ small eigenvalues, and this bound is sharp.

On the other hand, by a theorem of Zograf, if the number of cusps is much larger than the genus then there is necessarily at least 1 non-zero small eigenvalue. This provides a topological lower bound on the number of small eigenvalues for many cusped surfaces.

I will discuss joint work with Joe Thomas where we show that when n is much larger than g, any hyperbolic surface of signature (g, n) has at least $\propto \frac{2g+n}{\log(2g+n)}$ small eigenvalues.