Asymptotic optimization of Riesz means of Laplace eigenvalues on convex sets

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We consider the problem of optimizing Riesz means $\mathrm{Tr}(-\Delta - \Lambda_-^\gamma)$ of Laplace eigenvalues among convex sets in R^d with given measure. More precisely, we maximize Riesz means of Dirichlet eigenvalues and minimize Riesz means of Neumann eigenvalues. We are interested in the behavior of optimizers in the asymptotic regime where $\Lambda \to \infty$. In 2D we prove convergence in Hausdorff distance to a disk for any Riesz exponent $\gamma > 0$. We have similar results, either conditional or unconditional, in arbitrary dimension $d \geq 3$.

Our proofs combine uniform versions of Weyl asymptotics with the partially semiclassical analysis of degenerating convex sets.

The talk is based on joint work with Simon Larson.