RANDOM PERTURBATION OF TOEPLITZ MATRICES

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Toeplitz matrices form a rich and ubiquitous class of possibly non-normal matrices. Their asymptotic spectral analysis in high dimension is wellunderstood, as illustrated by the strong Szegő limit theorem for Toeplitz determinants. The spectra of these matrices are notoriously highly sensitive to small perturbations. In this talk, we explore the spectrum of a banded Toeplitz matrix perturbed by a random matrix in the asymptotic of high dimension. We show that the outlier eigenvalues are driven by a lowdimensional random analytic matrix field alongside an explicit deterministic matrix that captures the algebraic structure of the resonances responsible for the outlier eigenvalues. Along the way, we present new variations around the strong Szegő limit theorem. The talk is based on a joint work with Mireille Capitaine and François Chapon.