Spectral analysis of Dirac operators with a purely imaginary dislocation

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In this talk we present a complete spectral analysis of Dirac operators in 1D with non-Hermitian matrix potentials of the form $i\,\mathrm{sgn}(x)+V(x)$ where $V \in L^1$. For $V=0$ we compute explicitly the matrix Green function. This allows us to determine the spectrum, which is purely essential, and its different types. It also allows us to find sharp enclosures for the pseudospectrum and its complement, in all parts of the complex plane. Notably, this includes the instability region, corresponding to the interior of a band surrounding the real axis. Then, with the help of a Birman-Schwinger principle, we establish in precise manner how the spectrum and pseudospectrum change when $V \neq 0$, assuming the hypotheses $\|V\|_{L^1} < 1$ or $V \in L^1 \cap L^p$ where $p > 1$. We show that the essential spectra remain unchanged and that the ε-pseudospectrum stays close to the instability region for small ε. We determine sharp asymptotic estimates for the discrete spectrum, whenever V satisfies further conditions of decay at infinity. Finally, in one of our main findings, we give a complete description of the weakly-coupled model.

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