New minimal surfaces via equivariant eigenvalue optimization (part II)

Peter McGrath

North Carolina State University

April 8, 2024

In this follow-up to Daniel Stern's 4/1 talk, I discuss joint work with Karpukhin, Kusner, and Stern on the application of equivariant optimization of Laplace and Steklov eigenvalues on surfaces to constructions of embedded minimal surfaces in the 3-sphere and 3-ball. Riemannian metrics which maximize such normalized eigenvalues are known to give rise to branched minimal immersions by first eigenfunctions into spheres and balls, with codimension in general expected to grow with the topology of the surface. Nonetheless, there is a class ("Basic Reflection Surfaces", or BRS), of group actions on surfaces which allow each topological type and satisfy optimal eigenvalue bounds, ensuring that any branched minimal immersion by first eigenfunctions is a codimension-1 embedding, which doubles a minimal 2-sphere (or 2-disk) and has area less than 8π (or 2π). We show maximizing metrics can be found on each BRS, leading in particular to the existence of orientable, embedded minimal surfaces with free boundary in the 3-ball with arbitrary topological type, answering a question of Fraser and Li.