
AN OVERDETERMINED EIGENVALUE PROBLEM AND THE CRITICAL CATENOID CONJECTURE

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We consider the eigenvalue problem $\Delta^{\mathbb{S}^2}\xi + 2\xi = 0$ in Ω and $\xi = 0$ along $\partial\Omega$, being Ω the complement of a disjoint and finite union of smooth and bounded simply connected regions in the two-sphere \mathbb{S}^2 . Imposing that $\|\nabla\xi\|$ is locally constant along $\partial\Omega$ and that ξ has infinitely many maximum points, we are able to classify positive solutions as the rotationally symmetric ones. As a consequence, we obtain a characterization of the critical catenoid as the only embedded free boundary minimal annulus in the unit ball whose support function has infinitely many critical points.