On the geometric stability of Weyl's law and some applications to asymptotic spectral shape optimisation

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Denoting the Dirichlet and Neumann eigenvalues of a bounded Lipschitz domain $\Omega \subset \mathbb{R}^d$ by $0 < \lambda_1(\Omega) \leq \lambda_2(\Omega) \leq \lambda_3(\Omega) \leq \cdots$ and $0 = \mu_1(\Omega) \leq \mu_2(\Omega) \leq \mu_3(\Omega) \leq \cdots$ respectively, the well-known Weyl asymptotic formula asserts that

$$\lambda_k(\Omega) \sim \mu_k(\Omega) \sim W_d k^{2/d} |\Omega|^{-2/d}, \text{ as } k \to +\infty, \tag{1}$$

where $W_d > 0$ is the Weyl constant depending only on the dimension d and $|\Omega|$ is the volume of Ω.

In spectral shape optimisation problems, it is often desirable to have stability of the Weyl asymptotic formula for sequences of domains under some prescribed geometric conditions. For example, if one has a sequence of bounded Lipschitz domains $\Omega_k \subset \mathbb{R}^d$ of unit volume, under which further geometric conditions on the Ω_k is it true that

$$\lambda_k(\Omega_k) \sim \mu_k(\Omega_k) \sim W_d k^{2/d}, \text{ as } k \to +\infty? \tag{2}$$

One can ask similar questions more generally regarding the asymptotic behaviour of suprema/infima of Dirichlet/Neumann eigenvalues over collections of bounded Lipschitz domains.

We will discuss proofs of results in this spirit for bounded convex domains and consider some applications of these ideas in obtaining asymptotic results for spectral shape optimisation problems. In particular, we will consider minimising Dirichlet, Neumann and mixed Dirichlet-Neumann eigenvalues over collections of convex domains of a prescribed perimeter or diameter.