On the inner radius of nodal domains

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Consider a closed Riemannian manifold of dimension d. Let u be an eigenfunction of the Laplace-Beltrami operator with eigenvalue λ. Every connected component Ω of $u \neq 0$ is called a nodal domain of u. It follows from the Faber-Krahn inequality that $\mathrm{Vol}(\Omega) \geq C\lambda^{-d/2}$, where $C = C(M, g) > 0$. A refined question due to Leonid Polterovich is whether one can inscribe in Ω a ball of radius $C\lambda^{-1/2}$. The answer is positive in dimension two (M., 2006). In higher dimensions we show that this is true up to a logarithmic power factor: One can inscribe in Ω a ball of radius $C\lambda^{-1/2}(\log\lambda)^{-a_d}$. where a_d is a positive constant depending on dimension only. I will explain several ideas which go into the proof.

The talk is based on joint work with Philippe Charron.