Spectral Multiplicity and Nodal Domains of Torus-invariant Metrics

Samuel Lin

University of Oklahoma

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A classical result of Uhlenbeck states that for a generic Riemannian metric, the Laplace spectrum is simple, i.e., each eigenspace is real one-dimensional. On the other hand, manifolds with symmetries do not typically have simple spectra. If a compact Lie group G acts on a manifold as isometries, then each eigenspace is a representation of G, and hence, the spectrum cannot be simple in general. That each eigenspace is an irreducible representation for a generic G-invariant metric is a conjecture originating from quantum mechanics and atomic physics. In this work, we prove this conjecture for torus actions.

We also prove that for a generic torus-invariant metric, if u is a real-valued, non-invariant eigenfunction that vanishes on an orbit of the torus action, then the nodal set of u is a smooth hypersurface. This result provides a large class of Riemannian manifolds such that almost every eigenfunction has precisely two nodal domains. This starkly contrasts with previous results on the number of nodal domains for surfaces with ergodic geodesic flows. This is a joint project with Donato Cianci, Chris Judge, and Craig Sutton.