A variational method for functionals depending on eigenvalues

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Problems of optimisation of eigenvalues (or combination of eigenvalues) associated to an operator (Laplacian, Dirichlet-to-Neumann, etc) depending on a Riemannian metric on a differentiable manifold are often addressed in spectral geometry. For instance, they made it possible to build new minimal surfaces in the past decade. It is about finding the optimal bounds which depend on geometric and topologic data of the manifold and - when they are realized - extremal metrics.

We will explain a new variational method on functionals depending on eigenvalues. The classical methods do not work because these functionals are not C^1 at metrics which have eigenvalues with multiplicity bigger than 2 involved in the functional. We will then use in this context a subdifferential which provides the generalization of classical notions of gradient, critical points and Palais-Smale sequences.

As an application, we will explain how this new approach simplifies and unifies previous optimization results in dimension 2 and generalizes them in dimension higher than 3.