Towards an optimal spectral gap result for random compact hyperbolic surfaces

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The first non-zero Laplace eigenvalue of a hyperbolic surface, or its spectral gap, measures how well-connected the surface is: surfaces with a large spectral gap are hard to cut in pieces, have a small diameter and fast mixing times. For large hyperbolic surfaces (of large area or large genus g, equivalently), we know that the spectral gap is bounded above by $1/4 + o_g(1)$.

The aim of this talk is to present recent progress in my project with Nalini Anantharaman, where we aim to prove that most hyperbolic surfaces have a near-optimal spectral gap, i.e.:

$$\forall \epsilon > 0, \quad \lim_{g \to +\infty} \mathbb{P}_g \left(\lambda_1 \geq \frac{1}{4} - \epsilon \right) = 1.$$

Here, $\mathbb{P}_g$ denotes the Weil–Petersson probability measure on the moduli space of compact hyperbolic surfaces of genus g. This statement is analogous to Alon's 1986 conjecture for regular graphs, proven by Friedman in 2003.

I will present our approach, which is based on the trace method and shares many similarities with Friedman's work. I will explain some of the challenges that are encountered, and introduce new tools that we have developed in order to tackle them. So far, our methods allow us to obtain a spectral gap $2/9 - \epsilon$, hence improving on the bound $3/16 - \epsilon$ obtained by Wu–Xue and Lipnowski–Wright in 2021. To conclude the talk, we will discuss the final steps that need to be taken to obtain the optimal result.