Spectral tools for geometric degeneration

It is a classical question to study the way the (Laplace) spectrum of a geometric object behaves as the latter degenerates. Depending on the nature of the problem one is led to use either singular or analytic perturbation theory. I will present the basic difference between these two approaches and several well-known or more recent results. I will in particular emphasize the relation between these questions and (semiclassical) concentration properties and present applications to spectral simplicity of simple shapes. Most of this talk will be based on joint work with C. Judge.