Quantum ergodicity for periodic graphs

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Quantum ergodicity for graphs is a delocalization result which says the following. Suppose that a sequence of finite graphs Γ_N converges to some infinite graph Γ. Then *most* eigenfunctions $\psi_j^{(N)}$ of the adjacency matrix $\mathcal{A}_N$ on Γ_N become equidistributed on Γ_N when N gets large. More precisely, for most j, the probability measure $\sum_{v\in\Gamma_N} |\psi_j^{(N)}(v)|^2\delta_v$ approaches the uniform measure $\frac{1}{|\Gamma_N|}\sum_{v\in\Gamma_N}\delta_v$, in a weak sense. Potentials Q can sometimes be added, so that one now considers the eigenfunctions of $H_N = \mathcal{A}_N + Q_N$. Usually, the proof partially relies on certain nice properties of the infinite graph Γ. In particular, quantum ergodicity theorems have previously been established when Γ is a tree.

In this talk, I will present recent results of quantum ergodicity when Γ is invariant under translations of some basis of $\mathbb{Z}^d$, and the "fundamental block" is endowed a potential Q which is copied across the blocks, so that $H = \mathcal{A}_\Gamma + Q$ is a periodic Schrödinger operator. This framework includes $\Gamma = \mathbb{Z}^d$, the honeycomb lattice, strips, cylinders, etc.

I will first discuss the Bloch theorem and give some examples of its limitations, presenting along the way some very homogeneous graphs which violate quantum ergodicity. I will then discuss our main results, contrasting them with the tree case, give various examples of applications, and sketch the proof. An open problem concerning Schrödinger operators with a periodic potential on $\mathbb{Z}^d$, $d > 1$, will also be presented.

This talk is based on a joint work with Theo McKenzie (Harvard).