Convexity Properties for Harmonic Functions on Riemannian Manifolds

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In the 70's Almgren noticed that for a harmonic real-valued function defined on a ball, its L^2 -norm over a sub-sphere will have an increasing logarithmic derivative with respect to the radius of mentioned sphere. We examined similar integrals over a more general class of parameterized surfaces by studying harmonic functions defined on compact subdomains of Riemannian manifolds. The integrals over spheres are also generalized to level sets of a given function satisfying certain conditions. If we consider the L^2 norms over these level sets parametrized by a generalization of the radius, we again reproduce Almgren's convexity property. We will sketch the proof of this result and illustrate the usefulness of the convexity result by examining some explicit parameterized families of surfaces, e.g. geodesic spheres and ellipses.