On the polygonal Faber-Krahn inequality

Beniamin Bogosel

Ecole Polytechnique

May 9, 2022

It has been conjectured by Pólya and Szegö in 1951 that among n-gons with fixed area the regular one minimizes the first eigenvalue of the Dirichlet-Laplace operator. Despite its apparent simplicity, this result has only been proved for triangles and quadrilaterals. In this work we show that the proof of the conjecture can be reduced to finitely many certified numerical computations. Moreover, the local minimality of the regular polygon is reduced to a single validated numerical computation.

The steps of the proof strategy include the analytic computation of the Hessian matrix of the first eigenvalue, the stability of the Hessian with respect to vertex perturbations and analytic upper bounds for the diameter of an optimal set. Explicit a priori error estimates are given for the finite element computation of the eigenvalues of the Hessian matrix of the first eigenvalue associated to the regular polygon.

Results presented are obtained in collaboration with Dorin Bucur.