Maximization of Neumann Eigenvalues

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We discuss the maximization of the k-th eigenvalue of the Laplace operator with Neumann boundary conditions among domains of $\mathbb{R}^N$ with prescribed measure. We relax the problem to the class of (possibly degenerate) densities in $\mathbb{R}^N$ with prescribed mass and prove the existence of an optimal density. For $k = 1, 2$ the two problems are equivalent and the maximizers are known to be one and two equal balls, respectively. For $k \geq 3$ this question remains open, except in one dimension of the space where we prove that the maximal densities correspond to a union of k equal segments. This result provides sharp upper bounds for Sturm-Liouville eigenvalues and proves the validity of the Pólya conjecture in the class of densities in $\mathbb{R}$.

Based on the relaxed formulation, we provide numerical approximations of optimal densities for $k = 1, \ldots, 8$ in $\mathbb{R}^2$.

This is a joint work with E. Martinet and E. Oudet.