Stochastic homogenisation of high-contrast media

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Keywords: Homogenisation, Spectral Theory, High-contrast Random Composites.

We study the homogenisation problem for elliptic operators of the form $\mathcal{A}_\varepsilon = -\nabla A^\varepsilon \nabla$ with high-contrast random coefficients A^ε. In particular, we are interested in the behaviour of their spectra. We assume that on one of the components of the composite the coefficients A^ε are "of order one", the complimentary "soft" component consists of randomly distributed inclusions, whose size and spacing are of order $\varepsilon \ll 1$, and the values of A^ε on the inclusions are of order ε^2.

Our interest in high-contrast homogenisation problems is motivated by the band-gap structure of their spectra. From an intuitive point of view this phenomenon can be explained by looking at the "soft" inclusions as micro-resonators, which may dramatically amplify of completely block the propagation of waves in the medium, depending on the frequency. From a mathematically rigorous perspective, this was first analysed by Zhikov (2000, 2004) in the periodic setting.

Despite a vigorous activity in the field of periodic high-contrast homogenisation during the last two decades, the stochastic (random) high-contrast setting was largely overlooked, perhaps due to the technical challenges and and more complicated intuitive picture.

In this talk I will present our recent results for the stochastic high-contrast setting. First we will look at the homogenised operator $\mathcal{A}^{\mathrm{hom}}$ and describe its spectrum. Then I will give our main results on the convergence of the spectra of $\mathcal{A}_\varepsilon$ and a characterisation of the limit spectrum $\lim_{\varepsilon\to 0} \mathrm{Sp}(\mathcal{A}^{\mathrm{hom}})$. In contrast with the periodic setting, in the stochastic case the spectrum of the homogenised operator $\mathrm{Sp}(\mathcal{A}^{\mathrm{hom}})$ is, in general, a proper subset of $\lim_{\varepsilon\to 0} \mathrm{Sp}(\mathcal{A}^{\mathrm{hom}})$. We analyse the "additional" part of the spectrum - the difference between $\lim_{\varepsilon\to 0} \mathrm{Sp}(\mathcal{A}^{\mathrm{hom}})$ and $\mathrm{Sp}(\mathcal{A}^{\mathrm{hom}})$, and provide its *asymptotic* characterisation. Finally, I will touch upon the localisation of defect modes in the gaps of the limiting spectrum $\lim_{\varepsilon\to 0} \mathrm{Sp}(\mathcal{A}^{\mathrm{hom}})$ and our current and future work.