Bootstrap bounds on closed hyperbolic surfaces

James Bonifacio

University of Cambridge

March 14, 2022

In this talk I will discuss an approach for finding numerical bounds on the eigenvalues of the Laplace-Beltrami operator on closed orientable hyperbolic surfaces that is inspired by a method from physics called the conformal bootstrap. An example of such a bound is that the first nonzero eigenvalue, λ_1, must be less than 3.8388977. This bound is almost saturated by the Bolza surface, which is a genus-2 surface with $\lambda_1 \approx 3.8388873$. Similarly, this approach gives the bound $\lambda_1 \leq 2.678483$ for closed hyperbolic surfaces of genera at least 3, while the Klein quartic is a genus-3 surface with $\lambda_1 \approx 2.678$. These bounds were derived using numerical optimisation methods without rigorous error estimates, but they can also be made rigorous. I will discuss how to obtain these bounds, as well as some other applications of this approach.