Isoperimetric inequalities for the lowest Aharonov-Bohm eigenvalue of the Neumann and Steklov problems

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We discuss isoperimetric inequalities for the magnetic Laplacian on a bounded domain Ω endowed with an Aharonov-Bohm potential A with pole at a fixed point $x_0 \in \Omega$. Since A is harmonic on $\Omega \setminus \{x_0\}$, the magnetic field vanishes; the spectrum for the Neumann condition (or for the Steklov problem) reduces to that of the usual non-magnetic Laplacian, but only when the flux of the potential A around the pole is an integer. When the flux is not an integer the lowest eigenvalue is actually positive, and the scope of the talk is to show how to generalize the classical inequalities of Szëgo-Weinberger, Brock and Weinstock to the lowest eigenvalue of this particular magnetic operator, the model domain being a disk with the pole at its center. We consider more generally domains in the plane endowed with a rotationally invariant metric (which include the spherical and the hyperbolic case).