Towards Morse theory of dispersion relations

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The question of optimizing an eigenvalue of a family of self-adjoint operators that depends on a set of parameters arises in diverse areas of mathematical physics. Among the particular motivations for this talk are the Floquet-Bloch decomposition of the Schroedinger operator on a periodic structure, nodal count statistics of eigenfunctions of quantum graphs, and the minimal spectral partitions of domains and graphs. In each of these problems one seeks to identify and/or count the critical points of the eigenvalue with a given label (say, the third lowest) over the parameter space which is often known and simple, such as a torus.

Classical Morse theory is a set of tools connecting the number of critical points of a smooth function on a manifold to the topological invariants of this manifold. However, the eigenvalues are not smooth due to presence of eigenvalue multiplicities or "diabolical points". We rectify this problem for eigenvalues of generic families of finite-dimensional operators. The correct "Morse indices" of the problematic diabolical points turn out to be universal: they depend only on the total multiplicity at the diabolical point and on the relative position of the eigenvalue of interest in the eigenvalue group.

Based on a joint work with I.Zelenko.